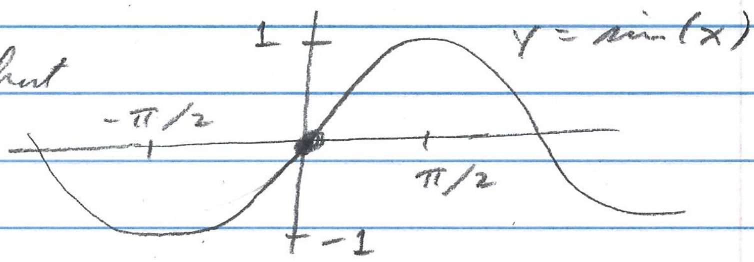
2.4.19 (a) Evaluate $\displaystyle\lim_{x \to 0^+} \frac{|\sin x|}{\sin x}$.

(b) Evaluate $\displaystyle\lim_{x \to 0^-} \frac{|\sin x|}{\sin x}$.

<u>Solution</u>

Recall that



So, when $x \to 0^+$ then x is "close to" 0 and positive. For such x-values, $\sin(x) > 0$ and so $|\sin x| = \sin x$.
When $x \to 0^-$ then x is "close to" 0 and negative. For such x-values, $\sin(x) < 0$ and so $|\sin(x)| = -\sin(x)$.

(a) $\displaystyle\lim_{x \to 0^+} \frac{|\sin x|}{\sin x} = \lim_{x \to 0^+} \frac{(\sin x)}{\sin x}$

since $|\sin x| = \sin x$ for $x \to 0^+$

$\displaystyle = \lim_{x \to 0^+} (1) = \boxed{1}.$

(b) $\displaystyle\lim_{x \to 0^-} \frac{|\sin x|}{\sin x} = \lim_{x \to 0^-} \frac{(-\sin x)}{\sin x}$

since $|\sin x| = -\sin x$ for $x \to 0^-$

$\displaystyle = \lim_{x \to 0^-} (-1) = \boxed{-1}.$ □