

2.4.33 Evaluate $\lim_{\theta \rightarrow 0} \frac{1 - \cos \theta}{\sin(2\theta)}$. Explain.

Solution

we have

$$\lim_{\theta \rightarrow 0} \frac{1 - \cos \theta}{\sin(2\theta)} = \lim_{\theta \rightarrow 0} \frac{1 - \cos \theta}{\sin(2\theta)} \left(\frac{2\theta}{2\theta} \right)$$

$$= \lim_{\theta \rightarrow 0} \frac{2\theta}{\sin(2\theta)} \frac{1 - \cos \theta}{2\theta} = \lim_{\theta \rightarrow 0} \frac{2\theta}{\sin(2\theta)} \lim_{\theta \rightarrow 0} \frac{1 - \cos \theta}{2\theta}$$

by the Product Rule, Theorem 2.1(4)

$$= \lim_{x \rightarrow 0} \frac{x}{\sin(x)} \frac{1}{2} \lim_{\theta \rightarrow 0} \frac{1 - \cos \theta}{\theta} \quad \begin{array}{l} \text{replacing } x \\ \text{with } 2\theta \text{ and} \end{array}$$

Observing that $\theta \rightarrow 0$ if and only if $x \rightarrow 0$,
and by the Constant Multiple Rule, Theorem 2.1(3)

$$= \frac{1}{2} \lim_{x \rightarrow 0} \frac{1}{(\sin x)/x} \lim_{\theta \rightarrow 0} \frac{1 - \cos \theta}{\theta}$$

$$= \frac{1}{2} \frac{1}{\lim_{x \rightarrow 0} (\sin x)/x} \lim_{\theta \rightarrow 0} \frac{1 - \cos \theta}{\theta} \quad \text{by the}$$

Quotient Rule, Theorem 2.1(5)

$$= \frac{1}{2} \frac{1}{(1)} (0) \quad \text{by Theorem 2.7 and Example 2.4.5(a)}$$

$$= \boxed{0}. \quad \square$$