

2.4.45 Evaluate  $\lim_{x \rightarrow 0} \frac{1 - \cos(3x)}{2x}$ . Explain.

Solution

We have

$$\lim_{x \rightarrow 0} \frac{1 - \cos(3x)}{2x} = \lim_{x \rightarrow 0} \frac{3}{2} \frac{1 - \cos(3x)}{3x}$$

$$= \lim_{\theta \rightarrow 0} \frac{3}{2} \frac{1 - \cos \theta}{\theta} \quad \text{letting } \theta = 3x \text{ and} \\ \text{noticing that } x \rightarrow 0 \\ \text{if and only if } \theta \rightarrow 0$$

$$= \frac{3}{2} \lim_{\theta \rightarrow 0} \frac{1 - \cos \theta}{\theta} \quad \text{by Theorem 2.1(3), the} \\ \text{Constant Multiple Rule}$$

$$= -\frac{3}{2} \lim_{\theta \rightarrow 0} \frac{\cos \theta - 1}{\theta} \quad \text{by Theorem 2.1(3), the} \\ \text{Constant Multiple Rule}$$

$$= -\frac{3}{2} (0) \quad \text{by Example 2.4.5(a)}$$

$$= \boxed{0}. \quad \square$$