2.4.53 Prove $\lim_{x \to 0^-} \frac{x}{|x|} = 1$.

Proof

First notice that $f(x) = \frac{x}{|x|}$ is defined on the interval $(0, \infty)$. Let $\varepsilon > 0$. We choose $\delta = 1$ (though any $\delta > 0$ will work). If $0 < x - 0 < \delta$, or $0 < x < 1$, then we have $|x| = x$ and

$$f(x) = \frac{x}{|x|} = \frac{x}{x} = 1$$ so that

$$|f(x) - 1| = |(1) - 1| = 0 < \varepsilon.$$

Hence, by the Formal Definition of One-Sided Limits, $L = 1$ and $\lim_{x \to 0} f(x) = \lim_{x \to 0} \frac{x}{|x|} = 1$. ■