2.5.21 At what points is $y = f(x) = \csc(2x)$ continuous?

Solution

Recall that $\csc\theta$ has domain all real numbers EXCEPT integer multiples of π; i.e. the domain is

$$\cdots \cup (-2\pi, -\pi) \cup (-\pi, 0) \cup (0, \pi) \cup (\pi, 2\pi) \cup \cdots$$
$$= \bigcup_{n=-\infty}^{\infty} (n\pi, (n+1)\pi).$$

We know that $\csc\theta$ is continuous on its domain by Theorem 2.5.1. So $\csc(2x)$ is continuous UNLESS $2x$ is an integer multiple of π. So set $2x = n\pi$ where $n \in \mathbb{Z}$ ← INTEGERS and this implies $x = \frac{n\pi}{2} = n\frac{\pi}{2}$.

So $\csc(2x)$ is continuous except at integer multiples of $\pi/2$.

OR $\csc(2x)$ is continuous for

$$x \in \bigcup_{n=-\infty}^{\infty} \left(n\frac{\pi}{2}, (n+1)\frac{\pi}{2}\right). \quad \square$$