

2.5.37

Find the limit and is the function continuous at the limit point for:

$$\lim_{t \rightarrow 0} \cos\left(\frac{\pi}{\sqrt{19-3\sec(2t)}}\right) ?$$

Solution

$$\begin{aligned} \text{First, } \cos\left(\frac{\pi}{\sqrt{19-3\sec(2(0))}}\right) &= \cos\left(\frac{\pi}{\sqrt{19-3(1)}}\right) \\ &= \cos\left(\frac{\pi}{\sqrt{16}}\right) = \cos\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}. \end{aligned}$$

The limit involves composition of functions! We start with the limit "inside" the inner most function and then use the definition of continuity to pull it out:

$$\frac{\sqrt{2}}{2} = \cos\left(\frac{\pi}{\sqrt{19-3\sec(2(0))}}\right)$$

$$= \cos\left(\frac{\pi}{\sqrt{19-3\sec\left(2\left(\lim_{t \rightarrow 0} t\right)\right)}}\right) \quad \begin{array}{l} \text{since} \\ \lim_{t \rightarrow 0} t = 0 \end{array}$$

$$= \cos\left(\frac{\pi}{\sqrt{19-3\sec\left(\lim_{t \rightarrow 0} 2t\right)}}\right) \quad \begin{array}{l} \text{by Theorem 2.1(3),} \\ \text{Constant Multiple} \\ \text{Rule} \end{array}$$

$$= \cos\left(\frac{\pi}{\sqrt{19-3\lim_{t \rightarrow 0} \sec(2t)}}\right) \quad \begin{array}{l} \text{since secant} \\ \text{is continuous} \\ \text{on its domain,} \\ \text{and } 0 \text{ is in the domain} \end{array}$$

$$= \cos \left(\frac{\pi}{\sqrt{\lim_{t \rightarrow 0} (19 - 3 \sec(2t))}} \right)$$

by Theorem 2.1 (2 and 3), Constant Multiple Rule and Difference Rule

$$= \cos \left(\frac{\pi}{\lim_{t \rightarrow 0} \sqrt{19 - 3 \sec(2t)}} \right)$$

by Theorem 2.1 (7), Root Rule

$$= \cos \left(\lim_{t \rightarrow 0} \left(\frac{\pi}{\sqrt{19 - 3 \sec(2t)}} \right) \right)$$

by Theorem 2.1 (5), Quotient Rule

$$= \lim_{t \rightarrow 0} \cos \left(\frac{\pi}{\sqrt{19 - 3 \sec(2t)}} \right)$$

since cosine is continuous everywhere.

Therefore,

$$\lim_{t \rightarrow 0} \cos \left(\frac{\pi}{\sqrt{19 - 3 \sec(2t)}} \right) = \boxed{\frac{\sqrt{2}}{2}}$$

We have shown that

$$\lim_{t \rightarrow 0} \cos\left(\frac{\pi}{\sqrt{19-3\sec(2t)}}\right) = \cos\left(\frac{\pi}{\sqrt{19-3\sec(2(0))}}\right)$$
$$= \frac{\sqrt{2}}{2}. \text{ So with } f(t) = \cos\left(\frac{\pi}{\sqrt{19-3\sec(2t)}}\right):$$

① $f(0)$ exists and $f(0) = \frac{\sqrt{2}}{2}$,

② $\lim_{t \rightarrow 0} f(t)$ exists and $\lim_{t \rightarrow 0} f(t) = \frac{\sqrt{2}}{2}$,

③ $\lim_{t \rightarrow 0} f(t) = f(0)$.

So, by the Test for Continuity, **YES**
this function is continuous at $t=0$.

□