




2.5.97

## Roots of a Cubic

Show that the equation  $x^3 - 15x + 1 = 0$  has three solutions in the interval  $[-4, 4]$ ,  
Solution

First, let  $f(x) = x^3 - 15x + 1$ . Then  $f$  is a polynomial function and so is continuous on  $[-4, 4]$  by Theorem 2.5.A. We apply the Intermediate Value Theorem (Theorem 2.11). Consider:

$x$	$f(x)$
-4	$(-4)^3 - 15(-4) + 1 = -3$
-3	$(-3)^3 - 15(-3) + 1 = 19$ 
-2	$(-2)^3 - 15(-2) + 1 = 23$
-1	$(-1)^3 - 15(-1) + 1 = 15$
0	$(0)^3 - 15(0) + 1 = 1$
1	$(1)^3 - 15(1) + 1 = -13$ 
2	$(2)^3 - 15(2) + 1 = -21$
3	$(3)^3 - 15(3) + 1 = -17$
4	$(4)^3 - 15(4) + 1 = 5$ 

As by the Intermediate Value Theorem,  $f(x) = 0$  for some  $c_1 \in (-4, -3)$ , for some  $c_2 \in (0, 1)$ , and for some  $c_3 \in (3, 4)$ .

□