2.6.17 Evaluate $\lim_{x \to \pm\infty} \left(\dfrac{7x^3}{x^3 - 3x^2 + 6x} \right)$.

Justify your computations.

Solution

We know $\lim_{x \to \pm\infty} \dfrac{1}{x} = 0$ by Example 2.6.1.

So we must use this! We have:

$$\lim_{x \to \pm\infty} \left(\frac{7x^3}{x^{3} - 3x^2 + 6x} \right) = \lim_{x \to \pm\infty} \left(\frac{7x^3}{x^3 - 3x^2 + 6x} \right) \left(\frac{1/x^3}{1/x^3} \right)$$

dividing numerator and denominator by the highest power of x in the denominator

$$= \lim_{x \to \pm\infty} \frac{7x^3/x^3}{x^3/x^3 - 3x^2/x^3 + 6x/x^3}$$

$$= \lim_{x \to \pm\infty} \frac{7}{1 - 3\left(\frac{1}{x}\right) + 6\left(\frac{1}{x^2}\right)}$$

$$= \frac{\lim_{x \to \pm\infty} (7)}{\lim_{x \to \pm\infty} (1) - 3 \boxed{\lim_{x \to \pm\infty} \left(\frac{1}{x}\right)} + 6 \left(\boxed{\lim_{x \to \pm\infty} \frac{1}{x}} \right)^2}$$

by Theorem 2.12 (1, 2, 4, 5, 6), the Sum Rule, Difference Rule, Constant Multiple Rule, Quotient Rule, and Power Rule (respectively)

$$= \frac{7}{1 - 3(0) + 6(0)} \quad \text{by Exercise 2.6.1}$$

$$= \frac{7}{1} = \boxed{7}. \qquad \square$$