2.6.65 Consider $y = \frac{1}{2x+4}$. Find all horizontal and vertical asymptotes. Then graph $y = f(x)$ in a such a way to reflect the asymptotic behavior.

Solution

For a horizontal asymptote, we consider $\lim_{x \to \pm\infty} \left(\frac{1}{2x+4}\right) = \lim_{x \to \pm\infty} \left(\frac{1}{2x+4}\right)\left(\frac{1/x}{1/x}\right)$

$= \lim_{x \to \pm\infty} \left(\frac{1/x}{2x/x + 4/x}\right) = \lim_{x \to \pm\infty} \left(\frac{1/x}{2+4(1/x)}\right)$

$$= \frac{\lim_{x \to \pm\infty} (1/x)}{2 + 4 \lim_{x \to \pm\infty} (1/x)} = \frac{(0)}{2+4(0)} = 0.$$

So, by definition, $y = f(x)$ has a $\boxed{\text{horizontal asymptote of } y = 0.}$

Now for vertical asymptotes. Notice that in this rational function the denominator $2x+4$ is 0 when $x = -2$ and the numerator is NOT 0 when $x = -2$. So, by Mr. Rob's Infinite Limits Theorem, $y = f(x) = \frac{1}{2x+4}$ has an infinite limit at $x = -2$.

So we consider the one-sided limits as $x \to -2^-$ and $x \to -2^+$ (and we make a "sign diagram").

For $x \to -2^-$ we have:

$$\frac{1}{2x+4} \Rightarrow \frac{(+)}{(-)} = - ,$$

so $\displaystyle\lim_{x \to -2^-} \left(\frac{1}{2x+4} \right) = -\infty.$

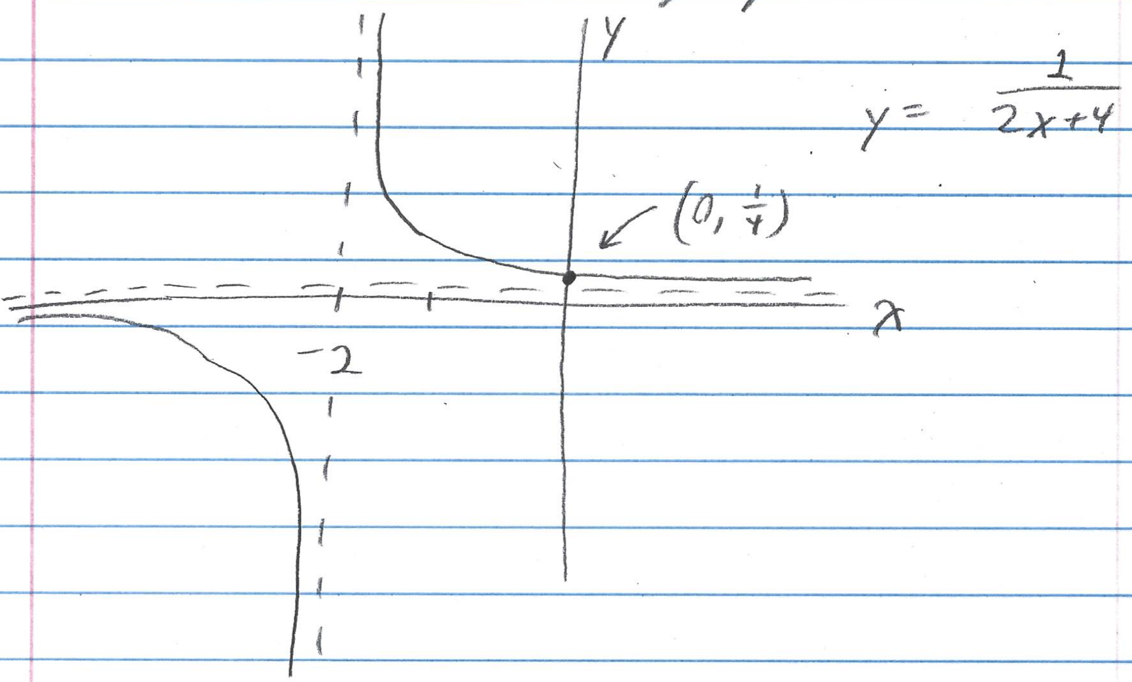
For $x \to -2^+$ we have:

$$\frac{1}{2x+4} \Rightarrow \frac{(+)}{(+)} = + ,$$

so $\displaystyle\lim_{x \to -2^+} \left(\frac{1}{2x+4} \right) = +\infty.$

Hence, $y = f(x)$ has a vertical asymptote at $x = -2$.

We then have the graph:



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