

3.1.21 Find the slope of the curve

$$y = f(x) = \frac{1}{x-1} \text{ at the point } x=3,$$

Solution

Recall that the slope m of the curve $y = f(x)$ at the point $P(x_0, f(x_0))$

is

$$m = \lim_{h \rightarrow 0} \frac{f(x_0+h) - f(x_0)}{h}.$$

We have $f(x) = \frac{1}{x-1}$ and $x_0 = 3$.

So

$$m = \lim_{h \rightarrow 0} \frac{f(x_0+h) - f(x_0)}{h} = \lim_{h \rightarrow 0} \frac{f(3+h) - f(3)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{\frac{1}{(3+h)-1} - \frac{1}{(3)-1}}{h}$$

$$= \lim_{h \rightarrow 0} \frac{1}{h} \left(\frac{1}{2+h} - \frac{1}{2} \right)$$

$$= \lim_{h \rightarrow 0} \frac{1}{h} \left(\frac{2 - (2+h)}{2(2+h)} \right)$$

$$\stackrel{F}{=} \lim_{h \rightarrow 0} \frac{1}{h} \left(\frac{-h}{2(2+h)} \right) \stackrel{C}{=} \lim_{h \rightarrow 0} \frac{-1}{2(2+h)}$$

$$\stackrel{S}{=} \frac{-1}{2(2+(0))} = \boxed{\frac{-1}{4}} \text{ since } \frac{-1}{2(2+h)} \text{ is a rational function (Thm 2.3). } \square$$