3.1, 29 An object is dropped from the top of a 100-m-high tower. Its height above ground after t sec is

$$100 - 4.9t^2 \text{ m.}$$

How fast is it falling 2 sec after it is dropped?

Solution

We have position ("height") as a function of time as $f(t) = 100 - 4.9t^2$ (position is measured in meters and time in seconds).

The question is: What is the instantaneous velocity when $t = 2$?

Well, instantaneous is the derivative of position:

$$\begin{pmatrix} \text{velocity} \\ \text{at } t_0 = 2 \end{pmatrix} = f'(t_0) = f'(2) = \lim_{h \to 0} \left(\frac{f(t_0+h) - f(t_0)}{h} \right) \Bigg|_{t_0 = 2}$$

$$= \lim_{h \to 0} \frac{f(2+h) - f(2)}{h} \quad \text{since } t_0 = 2$$

$$= \lim_{h \to 0} \frac{\left(100 - 4.9(2+h)^2\right) - \left(100 - 4.9(2)^2\right)}{h}$$

"FCS"

$$= \lim_{h \to 0} \frac{100 - 4.9\left(4 + 4h + h^2\right) - 100 + 4.9(4)}{h}$$

$$= \lim_{h \to 0} \frac{100 - 4.9(4) - 4.9(4)h - 4.9h^2 - 100 + 4.9(4)}{h}$$

$$= \lim_{h \to 0} \frac{-4.9(4)h - 4.9h^2}{h}$$

$$= \lim_{h \to 0} \frac{-4.9h(4-h)}{h}$$ Factoring

$$= \lim_{h \to 0} -4.9(4-h)$$ Canceling

$$= -4.9(4-(0))$$ Substituting
$-4.9(4-h)$ is a polynomial;
see Theorem 1.2.

$$= -4.9(4) = \boxed{-19.6 \text{ m/sec.}}$$

We might say that the object is FALLING at 19.6 m/sec. □