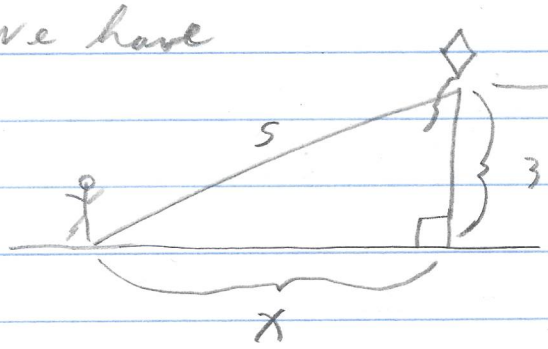
3.10.25 A girl flies a kite at a height of 300 ft, the wind carrying the kite horizontally away from her at a rate of 25 ft/sec. How fast must she let out string when the kite is 500 ft away from her?

Solution

(1) We have



Let s be the length of the string between the girl and the kite, and let x be the distance from the girl to the point on the ground directly under the kite.

(2) We are given $\frac{dx}{dt} = 25$ ft/sec.

(3) The question is $\frac{ds}{dt} = ?$ when $s = 500$ ft.

(4) The variables are related (by the Pythagorean Theorem) by $x^2 + 300^2 = s^2$.

(5) Differentiating implicitly with respect to t:

$$\frac{d}{dt}[x^2 + 300^2] = \frac{d}{dt}[s^2] \text{ or}$$

$$2x\left[\frac{dx}{dt}\right] + 0 = 2s\left[\frac{ds}{dt}\right]$$

or $\quad 2x\frac{dx}{dt} = 2s\frac{ds}{dt}$

or $\quad \frac{ds}{dt} = \frac{x}{s}\frac{dx}{dt}$.

⑥ Evaluate: WHEN $s = 500$ ft we have that $x^2 + 300^2 = s^2$ implies

$x^2 + 300^2 = 500^2$ or $x^2 = 500^2 - 300^2$

$= 250{,}000 - 90{,}000 = 160{,}000$ so that

$x = \sqrt{160{,}000} = 400$ ft (notice $x > 0$).

We have at this point in time that

$\frac{ds}{dt} = \frac{(400\text{ ft})}{(500\text{ ft})}(25\text{ ft/sec}) = \boxed{20\text{ ft/sec}}$ □