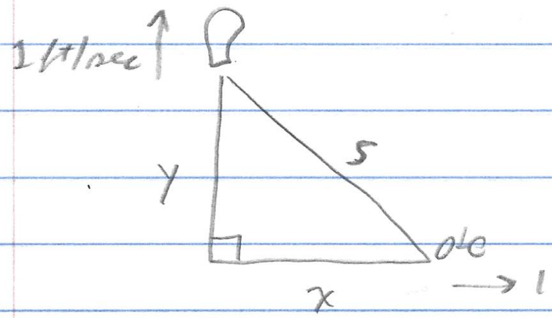
3.10.33 A balloon is rising vertically above a level, straight road at a constant rate of 1 ft/sec. Just when the balloon is 65 ft above the ground, a bicycle moving at a constant rate of 17 ft/sec passes under it. How fast is the distance s(t) between the bicycle and balloon increasing 3 sec later?

<u>Solution</u>

① Draw a picture:



Set up a coordinate system with the origin as the point directly under the balloon.

Let x be the distance from the origin to the bicycle, let y be the distance from the origin to the balloon (i.e., the height of the balloon), and let s be the distance from the balloon to the bicycle.

② We know

$$\frac{dy}{dt} = 1 \text{ ft/sec} \quad \text{and} \quad \frac{dx}{dt} = 17 \text{ ft/sec}.$$

③ The question is $\frac{ds}{dt} = ?$ at some point in time. At that point in time we have:

$x = (17 \text{ ft/sec})(3 \text{ sec}) = 51 \text{ ft}$

$y = (65 \text{ ft}) + (1 \text{ ft/sec})(3 \text{ sec}) = 68 \text{ ft}.$

By the Pythagorean Theorem, we have
$x^2 + y^2 = s^2$. So, WHEN $x = 51$ ft
and $y = 68$ ft then

$s^2 = (51 \text{ ft})^2 + (68 \text{ ft})^2 = 7{,}225 \text{ ft}^2$

or $s = \sqrt{7{,}225} \text{ ft} = 85 \text{ ft}.$

④ A relationship between the variables is given by the Pythagorean Theorem as $s^2 = x^2 + y^2$.

⑤ Differentiate implicitly with respect to time:

$$\frac{d}{dt}[s^2] = \frac{d}{dt}[x^2 + y^2]$$

$$2s\left[\frac{ds}{dt}\right] = 2x\left[\frac{dx}{dt}\right] + 2y\left[\frac{dy}{dt}\right]$$

or $$\frac{ds}{dt} = \frac{x}{s}\frac{dx}{dt} + \frac{y}{s}\frac{dy}{dt}$$

⑥ Evaluate: We want $\frac{ds}{dt}$ WHEN

$x = 51$ ft, $y = 68$ ft, $s = 85$ ft, $\frac{dx}{dt} = 17$ ft/sec, and

$\frac{dy}{dt} = 1$ ft/sec. So we have at this point in time

$$\frac{ds}{dt} = \frac{(51\text{ ft})}{(85\text{ ft})}(17\text{ ft/sec}) + \frac{(68\text{ ft})}{(85\text{ ft})}(1\text{ ft/sec})$$

$$= \frac{(51)(17)}{(85)} + \frac{68}{85}\text{ ft/sec}$$

$$= \frac{(51)(17)+68}{85}\text{ ft/sec} = \boxed{11\text{ ft/sec.}}\ \square$$