3.2.5 Consider $p(\theta) = \sqrt{3\theta}$. Find $p'(\theta)$, and $p'(1)$, $p'(3)$, $p'(2/3)$.

Solution

By definition $p'(\theta) = \lim_{h \to 0} \dfrac{p(\theta+h) - p(\theta)}{h}$.

So,

$$p'(\theta) = \lim_{h\to 0} \frac{p(\theta+h)-p(\theta)}{h} = \lim_{h\to 0} \frac{\sqrt{3(\theta+h)} - \sqrt{3\theta}}{h}$$

$$= \lim_{h\to 0} \left(\frac{\sqrt{3(\theta+h)} - \sqrt{3\theta}}{h}\right)\left(\frac{\sqrt{3(\theta+h)} + \sqrt{3\theta}}{\sqrt{3(\theta+h)} + \sqrt{3\theta}}\right)$$

$$= \lim_{h\to 0} \frac{\left(\sqrt{3(\theta+h)}\right)^2 - \left(\sqrt{3\theta}\right)^2}{h\left(\sqrt{3(\theta+h)} + \sqrt{3\theta}\right)}$$

$$= \lim_{h\to 0} \frac{(3\theta+3h)-(3\theta)}{h\left(\sqrt{3(\theta+h)} + \sqrt{3\theta}\right)} \overset{F}{=} \lim_{h\to 0} \frac{3h}{h\left(\sqrt{3(\theta+h)} + \sqrt{3\theta}\right)}$$

$$\overset{C}{=} \lim_{h\to 0} \frac{3}{\sqrt{3(\theta+h)} + \sqrt{3\theta}} \overset{S}{=} \frac{3}{\sqrt{3(\theta+(0))} + \sqrt{3\theta}}$$

$$= \frac{3}{\sqrt{3\theta} + \sqrt{3\theta}} = \boxed{\frac{3}{2\sqrt{3\theta}} = p'(\theta)}$$

Also, $\boxed{p'(1) = \dfrac{3}{2\sqrt{3(1)}} = \dfrac{3}{2\sqrt{3}}}$,

$$p'(3) = \frac{3}{2\sqrt{3(3)}} = \frac{3}{2\sqrt{9}} = \boxed{\frac{1}{2}}, \text{ and}$$

$$p'\left(\frac{2}{3}\right) = \frac{3}{2\sqrt{3\left(\frac{2}{3}\right)}} = \boxed{\frac{3}{2\sqrt{2}}}. \quad \square$$