

3.2.57 Derivative of  $-f$  Does knowing that a function  $f$  is differentiable at  $x = x_0$  tell you anything about the differentiability of the function  $-f$  at  $x = x_0$ ? Give reasons.

Solution

We have for  $-f$  that

$$\begin{aligned}(-f)'(x) &= \lim_{h \rightarrow 0} \frac{(-f(x+h)) - (-f(x))}{h} \\ &= \lim_{h \rightarrow 0} \frac{-(f(x+h) - f(x))}{h} = - \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\ &\quad \text{by Theorem 2.1(3), Constant Multiple Rule} \\ &= -(f'(x)).\end{aligned}$$

So the derivative of function  $(-f)$  is  $-(f')$ .

So if  $f$  is differentiable at  $x = x_0$  with value  $f'(x_0)$  then  $(-f)$

is differentiable at  $x = x_0$  and has value  $-(f'(x_0)) = -f'(x_0)$ .  $\square$