3.3.13 Differentiate $y = f(x) = (3 - x^2)(x^3 - x + 1)$.

Solution

(b) Well,

$$y = (3-x^2)(x^3 - x + 1) = 3x^3 - 3x + 3 - x^5 + x^3 - x^2$$

$$= -x^5 + 4x^3 - x^2 - 3x + 3.$$

So

$$y' = \frac{d}{dx}[y] = \frac{d}{dx}[-x^5 + 4x^3 - x^2 - 3x + 3]$$

$$= -[5x^4] + 4[3x^2] - [2x] - 3[1] + [0]$$

$$= \boxed{-5x^4 + 12x^2 - 2x - 3}.$$

(a) Also,

$$y' = \frac{d}{dx}[(3-x^2)(x^3 - x + 1)]$$

$$= \frac{d}{dx}[3-x^2](x^3 - x + 1) + (3-x^2)\frac{d}{dx}[x^3 - x + 1]$$

by the Product Rule (Theorem 3.3.6)

$$= [[0] - [2x]](x^3 - x + 1) + (3-x^2)[[3x^2] - [1] - [0]]$$

$$= \boxed{[-2x](x^3 - x + 1) + (3-x^2)[3x^2 - 1] = y'}$$

$$= (-2x^4 + 2x^2 - 2x) + (9x^2 - 3 - 3x^4 + x^2)$$

$$= -5x^4 + 12x^2 - 2x - 3.$$ ☺