3.3.69 Evaluate the limit $\lim_{x\to 1} \frac{x^{50}-1}{x-1}$ by first converting it to a derivative.

Solution

By the Alternative Formula for Derivatives (see Exercise 3.2.24 in the supplements)

$f'(x) = \lim_{z\to x} \frac{f(z)-f(x)}{z-x}$ or, interchanging x and z, $f'(z) = \lim_{x\to z} \frac{f(x)-f(z)}{x-z}$.

With $f(x) = x^{50}$ and $z = 1$ we then have

$$f'(1) = \lim_{x\to 1} \frac{x^{50}-(1)^{50}}{x-1} = \lim_{x\to 1} \frac{x^{50}-1}{x-1}.$$

Since $f(x) = x^{50}$ implies $f'(x) = 50x^{49}$ and so $f'(1) = 50(1)^{49} = 50$. Therefore

$$\boxed{\lim_{x\to 1} \frac{x^{50}-1}{x-1} = f'(1) = 50.}$$ □