3.4.5 Consider the position function

$$s(t) = \frac{25}{t^2} - \frac{5}{t} \quad \text{for } t \in [1, 5].$$

(a) Find the displacement and average velocity over the interval.

(b) Find speed and acceleration at the endpoints.

(c) When does the body change direction?

Solution

(a) Well, the displacement is

$$\Delta s = s(5) - s(1)$$

$$= \left(\frac{25}{(5)^2} - \frac{5}{(5)}\right) - \left(\frac{25}{(1)^2} - \frac{5}{(1)}\right) \text{ m}$$

$$= \frac{25}{25} - 1 - 25 + 5 = \boxed{-20 \text{ m}}$$

Next,

$$\begin{pmatrix}\text{average}\\ \text{velocity}\end{pmatrix} = v_{av} = \frac{\Delta s}{\Delta t} = \frac{-20 \text{ m}}{(5-1) \text{ sec}} = \boxed{-5 \text{ m/sec}}$$

(b) We velocity $v(t) = s'(t)$ and acceleration $a(t) = v'(t) = s''(t)$. We have

$$s(t) = \frac{25}{t^2} - \frac{5}{t} = 25t^{-2} - 5t^{-1},$$

$-2-1=-3$

so $v(t) = s'(t) = 25\left[-2t^{-3}\right] - 5\left[-t^{-2}\right]$

$-1-1=-2$

$$= -50t^{-3} + 5t^{-2} = \frac{-50}{t^3} + \frac{5}{t^2} \text{ m/sec}$$

Next, acceleration $a(t) = v'(t) = s''(t)$

is $a(t) = \frac{d}{dt}[v(t)] = \frac{d}{dt}\left[-50t^{-3} + 5t^{-2}\right]$

$-3-1=-4$ $= -50\left[-3t^{-4}\right] + 5\left[-2t^{-3}\right]$

$-2-1=-3$ $= \frac{150}{t^4} - \frac{10}{t^3}$ m/sec^2.

So the speed at the endpoints $t_0 = 1$ and $t_1 = 5$ is

$$|v(t_0)| = |v(1)| = \left|\frac{-50}{(1)^3} + \frac{5}{(1)^2}\right|$$

$$= |-50+5| = \boxed{45 \text{ m/sec}}$$

and $$|v(t_1)| = |v(5)| = \left|\frac{-50}{(5)^3} + \frac{5}{(5)^2}\right|$$

$$= \left|\frac{-50}{125} + \frac{5}{25}\right| = \left|\frac{-2}{5} + \frac{1}{5}\right| = \boxed{\frac{1}{5} \text{ m/sec}}$$

The acceleration at the endpoints is

$$a(t_0) = a(1) = \frac{150}{(1)^4} - \frac{10}{(1)^3}$$

$$= 150 - 10 = \boxed{140 \text{ m/sec}^2}$$

$$a(t_1) = a(5) = \frac{150}{(5)^4} - \frac{10}{(5)^3}$$

$$= \frac{150}{625} - \frac{10}{125} = \frac{3}{25} - \frac{2}{25} = \boxed{\frac{1}{25} \text{ m/sec}^2}$$

(c) When does the body change direction?
Well, when the object has positive velocity it is moving "forward" (or maybe "up") AND when the velocity is negative then the object is moving "backward") (or maybe "down").
SO, if the object changes direction then the velocity must be 0.
Hence we consider the formula $v(t) = 0$ or

$$\frac{-50}{t^3} + \frac{5}{t^2} = 0 \quad \text{for } t \in [1, 5].$$

for we need $\frac{5}{t^2} = \frac{50}{t^3}$ or $5t^3 = 50t^2$
or $t^3 = 10t^2$ or $t = 10$ sec (since $t \neq 0$).

NO, in the interval $t \in [1, 5]$ the velocity is not 0 and the function does not change direction!
(BUT, at time $t = 10$ sec it does have velocity 0 and ...). □