3.5.32 Find dp/dq for $p = \dfrac{3q + \tan q}{q \sec q}$.

Solution

By the Derivative Quotient Rule (Theorem 3.3.H) and the Derivative Product Rule (Theorem 3.3.G) we have

$$\frac{dp}{dq} = \frac{[3 + \sec^2 q](q \sec q) - (3q + \tan q)[[1]\sec q + (q)[\sec q \tan q]]}{(q \sec q)^2}$$

$$= \Big\{(3 + \sec^2 q)(q \sec q) - 3q \sec q - 3q^2 q \sec q \tan q + \tan q \sec q + q \sec q \tan q\Big\} / (q^2 \sec^2 q)$$

$$= \frac{(3 + \sec^2 q)q - 3q - 3q^3 \tan q + \tan q + q \tan q}{q^2 \sec q}$$

$$= \frac{q \sec^2 q - 3q^3 \tan q + \tan q + q \tan q}{q^2 \sec q}. \qquad \square$$