3.5.49 Evaluate $\lim_{x \to 2} \sin\left(\frac{1}{x} - \frac{1}{2}\right)$.

Solution

We have

$$0 = \sin(0) = \sin\left(\frac{1}{(2)} - \frac{1}{2}\right)$$

$$= \sin\left(\lim_{x \to 2}\left(\frac{1}{x}\right) - \frac{1}{2}\right) \text{ since } \lim_{x \to 2}\left(\frac{1}{x}\right) = \frac{1}{2}$$

since $\frac{1}{x}$ is a rational function; Theorem 2.3

$$= \sin\left(\lim_{x \to 2}\left(\frac{1}{x} - \frac{1}{2}\right)\right) \text{ by the Difference Rule for Limits (Theorem 2.1(2))}$$

$$= \lim_{x \to 2} \sin\left(\frac{1}{x} - \frac{1}{2}\right) \text{ since the sine function is continuous at 0; by Theorem 2.10 "Limits of Continuous Functions".}$$

So $\lim_{x \to 2} \sin\left(\frac{1}{x} - \frac{1}{2}\right) = \boxed{0}$. □