3.6.33 Differentiate $y = (4x+3)^4 (x+1)^{-3}$.

Solution

Let's apply the Product Rule (Theorem 3.3.6) to this product of compositions:

$$y' = \left[4(4x+3)^3 [4]\right](x+1)^{-3} + (4x+3)^4 \left[-3(x+1)^{-4}[1]\right]$$

$$= 16(4x+3)^3 (x+1)^{-3} - 3(4x+3)^4 (x+1)^{-4}$$

$$= (4x+3)^3 (x+1)^{-4} \left(16(x+1) - 3(4x+3)\right)$$

$$= (4x+3)^3 (x+1)^{-4} (16x + 16 - 12x - 9)$$

$$= \frac{(4x+3)^3 (4x+7)}{(x+1)^4}. \quad \square$$