3.6.97 (a) Find the tangent to the curve

$$y = 2\tan\left(\frac{\pi x}{4}\right) \text{ at } x = 1.$$

Solution

Well, y' gives the slopes of tangent lines as a function of x. We have

$$y' = 2\sec^2\left(\frac{\pi x}{4}\right)\left[\frac{\pi}{4}\right]$$

$$= \frac{\pi}{2}\sec^2\left(\frac{\pi x}{4}\right).$$

So the slope of the curve at $x = 1$ is $y'(1) = \frac{\pi}{2}\sec^2\left(\frac{\pi(1)}{4}\right) = \frac{\pi}{2}\sec^2\left(\frac{\pi}{4}\right)$

$$= \frac{\pi}{2}\frac{1}{\cos^2(\pi/4)} = \frac{\pi}{2}\frac{1}{(\sqrt{2}/2)^2}$$

$$= \frac{\pi}{2}\frac{1}{2/4} = \pi = m.$$

The point on the curve when $x = 1$ is (x, y) where $y = 2\tan\left(\frac{\pi(1)}{4}\right) = 2\tan\left(\frac{\pi}{4}\right) = 2$;

or $(x, y) = (1, 2)$. So the equation of the line is $y - y_1 = m(x - x_1)$ or

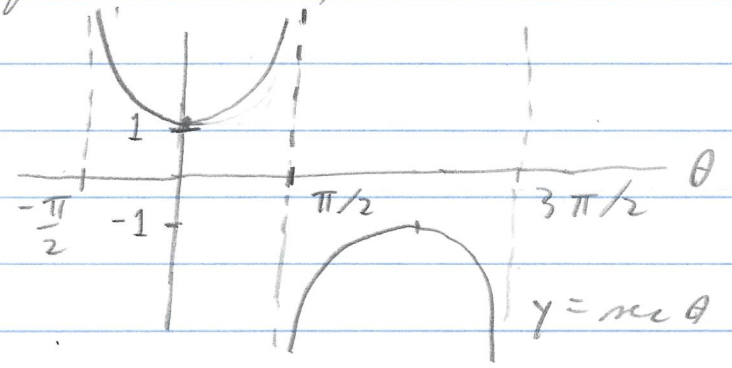
$y - (2) = \pi(x - (1))$ or $y - 2 = \pi x - \pi$

or $\boxed{y = \pi x - \pi + 2}$ □

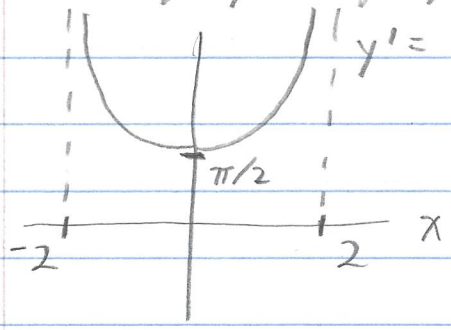
(6) What is the smallest value the slope of the curve can ever have on the interval $(-2,2)$?

Solution

Notice $y' = \frac{\pi}{2}\sec^2\left(\frac{\pi x}{4}\right)$, so the question is to minimize the slope of the curve y' for $x \in (-2, 2)$. Notice the graph of $y = \sec\theta$:



For $x \in (-2, 2)$ we have $\frac{\pi x}{4} \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, so the graph of y' over $(-2,2)$ is:



So y' is a minimum of $\pi/2$ at $x = 0$. That is, the smallest the slope of the curve can be on $(-2,2)$ is $\boxed{\pi/2.}$ □