3.7.25 Find dy/dx and dy^2/dx^2 for

$$2\sqrt{y} = x - y.$$

Solution

Well, we have

$$2y^{1/2} = x - y$$ and so

$$\frac{d}{dx}\left[2y^{1/2}\right] = \frac{d}{dx}\left[x - y\right],$$ or

$$2\left(\frac{1}{2}y^{-1/2}\left[\frac{dy}{dx}\right]\right) = 1 - \frac{dy}{dx},$$ or

$$\frac{1}{\sqrt{y}}\frac{dy}{dx} = 1 - \frac{dy}{dx},$$ or

$$\frac{1}{\sqrt{y}}\frac{dy}{dx} + \frac{dy}{dx} = 1$$ or

$$\frac{dy}{dx}\left(\frac{1}{\sqrt{y}} + 1\right) = 1$$ or

$$\frac{dy}{dx} = \frac{1}{\frac{1}{\sqrt{y}} + 1} = \frac{1}{\frac{1}{\sqrt{y}} + 1}\left(\frac{\sqrt{y}}{\sqrt{y}}\right),$$ or

$$\boxed{\frac{dy}{dx} = \frac{\sqrt{y}}{1 + \sqrt{y}}}$$

Next, $\frac{d}{dx}\left[\frac{dy}{dx}\right] = \frac{d}{dx}\left[\frac{\sqrt{y}}{1+\sqrt{y}}\right]$ a

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left[\frac{y^{1/2}}{1+y^{1/2}}\right]$$

$$= \frac{\left[\frac{1}{2}y^{-1/2}\left[\frac{dy}{dx}\right]\right](1+y^{1/2}) - (y^{1/2})\left[0+\frac{1}{2}y^{-1/2}\left[\frac{dy}{dx}\right]\right]}{(1+y^{1/2})^2}$$

$$= \frac{\frac{1}{2}\frac{1}{\sqrt{y}}\left(\frac{dy}{dx}\right)(1+\sqrt{y}) - \sqrt{y}\left(\frac{1}{2}\frac{1}{\sqrt{y}}\frac{dy}{d\cancel{x}}\right)}{(1+y^{1/2})^2}$$

$$= \frac{\frac{1}{2}\frac{1}{\sqrt{y}}\left(\frac{y^{1/2}}{1+y^{1/2}}\right)(1+\sqrt{y}) - y^{1/2}\left(\frac{1}{2}\frac{1}{y^{1/2}}\frac{y^{1/2}}{1+y^{1/2}}\right)}{(1+y^{1/2})^2}$$

$$= \boxed{\frac{\frac{1}{2}\left(\frac{y^{1/2}}{1+y^{1/2}}\right)\left(\frac{1}{y^{1/2}}(1+y^{1/2}) - 1\right)}{(1+y^{1/2})^2}}$$

$$= \frac{1 - y^{1/2}/(1+y^{1/2})}{2(1+y^{1/2})^2} = \frac{(1+y^{1/2}) - y^{1/2}}{2(1+y^{1/2})^3}$$

$$= \frac{1}{2(1+\sqrt{y})^3}. \quad \square$$