

3.8.33 Find dy/dx for $y = \ln\left(\frac{1}{x\sqrt{x+1}}\right)$.

Solution

$$\begin{aligned} \text{Notice that } y &= \ln\left(\frac{1}{x\sqrt{x+1}}\right) = -\ln(x\sqrt{x+1}) \\ &= -\ln(x) - \ln\sqrt{x+1} = -\ln(x) - \ln(x+1)^{1/2} \\ &= -\ln(x) - \frac{1}{2}\ln(x+1). \quad \text{So} \end{aligned}$$

$$\frac{dy}{dx} = \frac{d}{dx} \left[-\ln(x) - \frac{1}{2}\ln(x+1) \right]$$

$$= -\left[\frac{1}{x}\right] - \frac{1}{2} \left[\frac{1}{x+1} \overset{\curvearrowright}{[1]} \right]$$

$$= \boxed{\frac{-1}{x} - \frac{1}{2(x+1)}}$$

$$= \frac{-2(x+1) - x}{2x(x+1)} = \frac{-3x-2}{2x(x+1)} \quad \square$$