

3.8.41 Use logarithmic differentiation to find dy/dx
for $y = \sqrt{x(x+1)}$

Solution

$$\text{We have } \ln y = \ln \sqrt{x(x+1)} = \ln (x(x+1))^{1/2}$$
$$= \frac{1}{2} \ln (x(x+1)) = \frac{1}{2} (\ln x + \ln(x+1))$$

$$= \frac{1}{2} \ln(x) + \frac{1}{2} \ln(x+1). \quad \text{So}$$

$$\frac{d}{dx} [\ln y] = \frac{d}{dx} \left[\frac{1}{2} \ln(x) + \frac{1}{2} \ln(x+1) \right]$$

$$\frac{1}{y} \left[\frac{dy}{dx} \right] = \frac{1}{2} \left[\frac{1}{x} \right] + \frac{1}{2} \left[\frac{1}{x+1} \right]$$

$$\text{or } \frac{dy}{dx} = y \left(\frac{1}{2x} + \frac{1}{2(x+1)} \right)$$

$$= \boxed{\sqrt{x(x+1)} \left(\frac{1}{2x} + \frac{1}{2(x+1)} \right)} \quad \square$$