

4.1.53

Consider $y = f(x) = \ln(x+1) - \tan^{-1}(x)$.

Find the domain and the critical points.

Solution

Notice that $\ln(x+1)$ is only defined for $x+1 > 0$ or $x > -1$ or $x \in (-1, \infty)$.

Next, $\tan^{-1}(x)$ is defined for all $x \in \mathbb{R}$ (i.e., all real x). Hence,

the domain of f is $(-1, \infty)$.

The derivative of f is

$$f'(x) = \frac{1}{x+1} [1] - \frac{1}{1+x^2} = \frac{1}{1+x} - \frac{1}{1+x^2}$$

$$= \frac{(1+x^2) - (1+x)}{(1+x)(1+x^2)} = \frac{x^2 - x}{(1+x)(1+x^2)}$$

$$= \frac{x(x-1)}{(1+x)(1+x^2)}$$

Notice that $f'(0) = 0$ and $f'(1) = 0$.

However, there are not points in $(-1, \infty)$ where $f'(x)$ is undefined. So

the critical points are $x = 0$ and $x = 1$ where the derivative is 0. □