4.2.1 Consider $f(x) = x^2 + 2x - 1$ on $[0,1]$. Find $c \in (0,1)$ such that

$$f'(c) = \frac{f(b) - f(a)}{b - a} \text{ where } [a,b] = [0,1].$$

Solution

Notice that $f(x) = x^2 + 2x - 1$ is continuous on $[0,1]$ (since it is a polynomial functions) and $f'(x) = 2x + 2$ so that f is differentiable on $(0,1)$ (since it is a polynomial function), so the hypotheses of the Mean Value Theorem are satisfied and such a value c must exist!

Next, we want

$$f'(c) = \frac{f(b) - f(a)}{b - a} \text{ or}$$

$$2(c) + 2 = \frac{((1)^2 + 2(1) - 1) - ((0)^2 + 2(0) - 1)}{1 - 0}$$

or $2c + 2 = 3$ or $2c = 1$ or $\boxed{c = 1/2}$. $\square$