4.3.23 Consider $f(\theta) = 3\theta^2 - 4\theta^3$. Where is f INC/DEC? What are extrema? Graph it!

Solution

Well, for INC/DEC we need

$f'(\theta) = 6\theta - 12\theta^2 = 6\theta(1-2\theta)$.

So $\theta = 0$ and $\theta = 1/2$ are critical points because $f' = 0$ there. We perform a SIGN TEST on f':

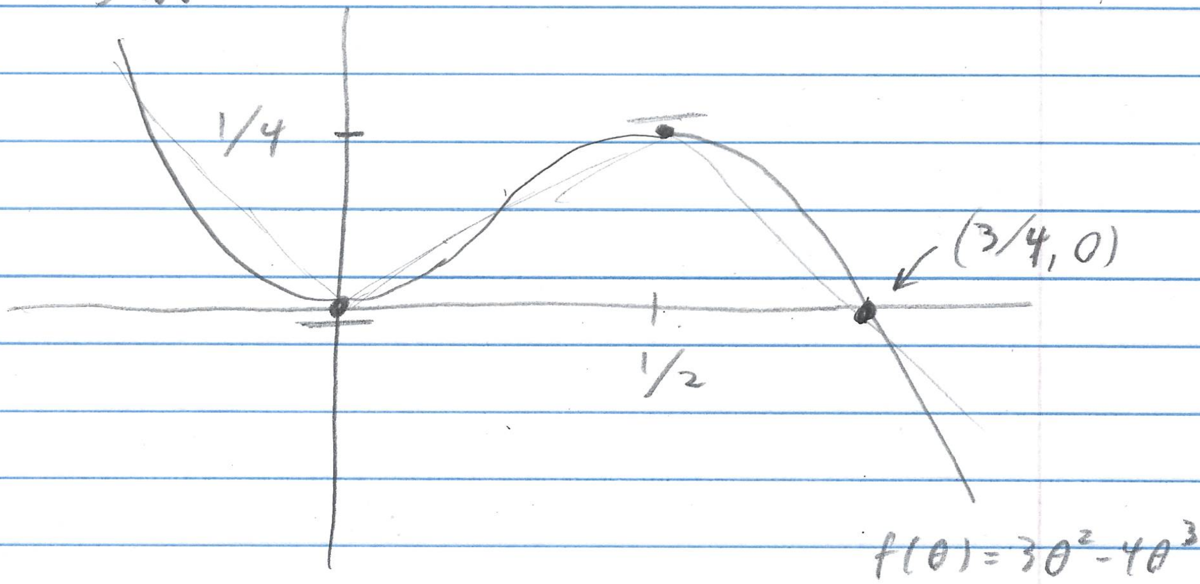
	$(-\infty, 0)$	$(0, 1/2)$	$(1/2, \infty)$
test value k	-1	$1/4$	1
$f'(k)$	$6(-1)-12(-1)^2 = -18$	$6(\frac{1}{4})-12(\frac{1}{4})^2 = \frac{3}{2}-\frac{3}{4}=\frac{3}{4}$	$6(1)-12(1)^2$ -6
$f'(\theta)$	$-$	$+$	$-$
$f(\theta)$	DEC	INC	DEC

SO, f is DEC on $(-\infty, 0] \cup [1/2, \infty)$

f is INC on $[0, 1/2]$.

By The First Derivative Test for Local Extrema (Theorem 4.3.A), f has a local MIN at $\theta = 0$ of $f(0) = 3(0)^2 - 4(0)^3 = 0$ and f has a local MAX at $\theta = 1/2$ of $f(1/2) = 3(1/2)^2 - 4(1/2)^3 = \frac{3}{4} - \frac{1}{2} = \frac{1}{4}$.

We have



$\boxed{\text{Notice that } f \text{ has no absolute extrema.}}$ $\square$