4.3.41 Consider $f(x) = e^{2x} + e^{-x}$. Where is f INC/DEC? What are extrema?

Solution

We have $f'(x) = e^{2x}[2] + e^{-x}[-1]$

$= 2e^{2x} - e^{-x} = \frac{2e^{3x}}{e^x} - \frac{1}{e^x} = \frac{2e^{3x}-1}{e^x}$.

So we have a critical point when $2e^{3x} - 1 = 0$ or $e^{3x} = 1/2$ or $\ln(e^{3x}) = \ln(1/2)$ or $3x = \ln(2^{-1})$ or $x = -\ln(2)/3$. We perform a SIGN TEST on f':

	$\left(-\infty, \frac{-\ln(2)}{3}\right)$	$\left(\frac{-\ln(2)}{3}, \infty\right)$
TEST VALUE h	$-\ln(2)$	0
$f'(h)$	$\frac{2e^{3(-\ln 2)}-1}{e^{(-\ln 2)}} = \frac{2(2^{-3})-1}{2^{-1}} = \frac{-3/4}{1/2} = \frac{-3}{2}$	$\frac{2e^{3(0)}-1}{e^0} = 1$
$f'(x)$	$-$	$+$
$f(x)$	DEC	INC

So f is DEC on $(-\infty, -(\ln 2)/3]$ and f is INC on $[-(\ln 2)/3, \infty)$

By The First Derivative Test for Local Extrema (Theorem 4.3.A), f has a local MIN at $x = -(\ln 2)/3$ of

$$e^{2(-(\ln 2)/3)} + e^{-(-(\ln 2)/3)} = 2^{-2/3} + 2^{1/3}$$

$$= \frac{1}{2^{2/3}} + \frac{2}{2^{2/3}} = \frac{3}{2^{2/3}}.$$

From the INC/DEC information we see that f has an absolute MIN at $x = -(\ln 2)/3$ of $3/2^{2/3}$ and f has no absolute MAX.

$\square$