4.3.73 Sketch the graph of a continuous function $y = g(x)$ such that:

(a) $g(2) = 2$, $0 < g' < 1$ for $x < 2$, $g'(x) \to 1^-$ as $x \to 2^-$, $-1 < g' < 0$ for $x > 2$, and $g'(x) \to -1^+$ as $x \to 2^+$.

Solution

Well, we have

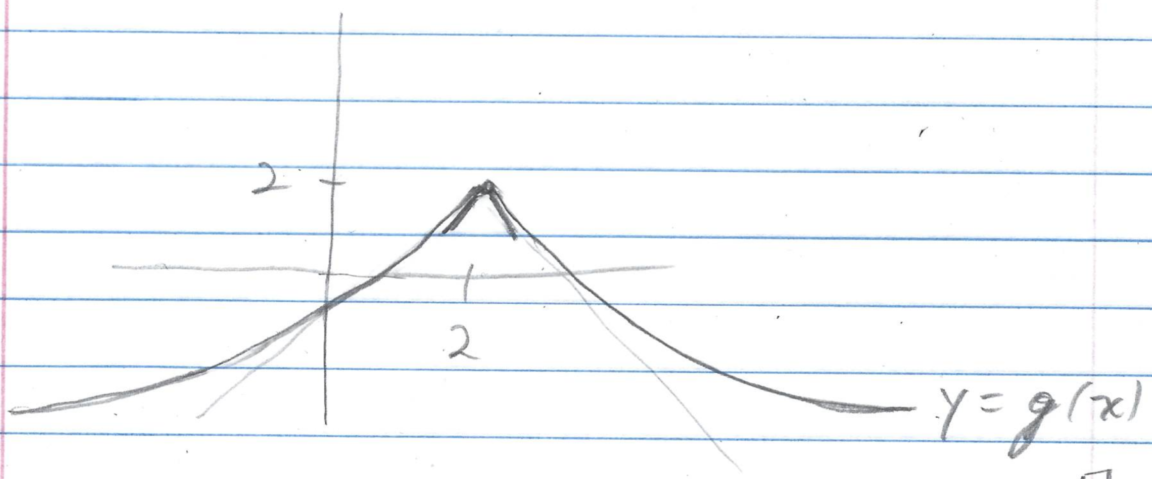
	$(-\infty, 2)$	$(2, \infty)$
$g'(x)$	+	−
$g(x)$	INC	DEC

By the First Derivative Test for Local Extrema (Theorem 4.3.A) we see that g has a local MAX at $x = 2$ of $g(2) = 2$.

Notice $g'(x) \to 1^-$ as $x \to 2^-$ OR $\lim_{x \to 2^-} g'(x) = 1$, and $g'(x) \to -1^+$ as $x \to 2^+$ OR $\lim_{x \to 2^+} g'(x) = -1$.

SO, $\lim_{x \to 2} g'(x)$ does not exist by Theorem 2.6 (Relation Between One-Sided and Two-Sided Limits).

So the graph is



$\square$

(b) $g(2) = 2$, $g' < 0$ for $x < 2$, $g'(x) \to -\infty$ as $x \to 2^-$; $g' > 0$ for $x > 2$, and $g'(x) \to \infty$ as $x \to 2^+$.

Solution

Well, we have

	$(-\infty, 2)$	$(2, \infty)$
$g'(x)$	$-$	$+$
$g(x)$	DEC	INC

By the First Derivative Test for Local Extrema (Theorem 4.3.A) we see that g has a local MIN at $x = 2$ of $g(2) = 2$.

Notice $g'(x) \to -\infty$ as $x \to 2^-$ OR $\lim_{x \to 2^-} g'(x) = -\infty$, and $g'(x) \to \infty$ as $x \to 2^+$ OR $\lim_{x \to 2^+} g'(x) = \infty$. So the graph of $y = g(x)$ has a "cusp" at $x = 2$ (see Section 3.2).

So the graph is