4.4.19 Consider $y = f(x) = 4x^3 - x^4 = x^3(4-x)$.
Find where f is INC/DEC, extrema,
where f is CU/CD, and points of inflection.

Solution

Now $f'(x) = 12x^2 - 4x^3 = 4x^2(3-x)$, so
$x=0$ and $x=3$ are critical points because
$f'=0$ there. So we consider

	$(-\infty, 0)$	$(0, 3)$	$(3, \infty)$
TEST VALUE k	-1	1	4
$f'(k)$	$4(-1)^2(3-(-1))$ $=16$	$4(1)^2(3-(1))$ $=8$	$4(4)^2(3-(4))$ $=-64$
$f'(x)$	$+$	$+$	$-$
$f(x)$	INC	INC	DEC

So f is INC on $(-\infty, 3]$ and f is DEC
on $[3, \infty)$. By the First Derivative Test
for Local Extrema (Theorem 4.3.A), f has
a local MAX at $x=3$ of $f(3) = 4(3)^3 - (3)^4$
$= 108 - 81 = 27$.

Next, for concavity consider
$f''(x) = 24x - 12x^2 = 12x(2-x)$. So f has
potential points of inflection at $x=0$ and $x=2$
since $f''=0$ there. So we consider ...

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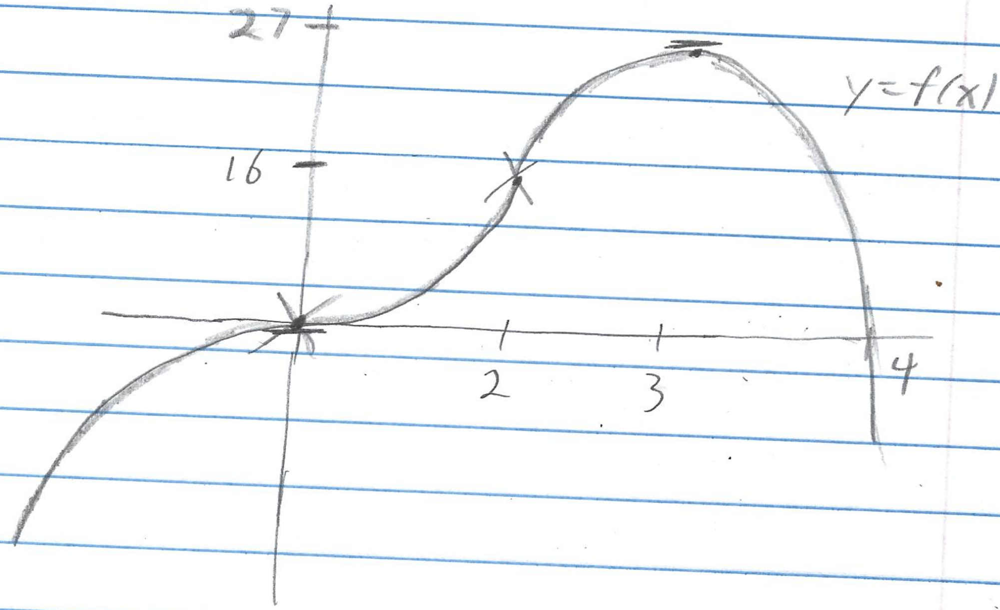
	$(-\infty, 0)$	$(0,2)$	$(2,\infty)$
TEST VALUE h	-1	1	3
$f''(h)$	$24(-1)-12(-1)^2 = -36$	$24(1)-12(1)^2 = 12$	$24(3)-12(3)^2 = -36$
$f''(x)$	$-$	$+$	$-$
$f(x)$	CD	CU	CD

So f is CD on $(-\infty,0)\cup(2,\infty)$ and f is CU on $(0,2)$.

Hence $x=0$ is a critical point where $f(0)=0$, and $x=2$ is a critical point where $f(2)=4(2)^3-(2)^4 = 32-16=16$.

The graph is



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