4.4.25 Consider $y = f(x) = \sqrt{3}\,x - 2\cos x$, $x \in [0, 2\pi]$. Find where f is INC/DEC, extrema, where f is CU/CD, and points of inflection!

<u>Solution</u>

Now $f'(x) = \sqrt{3} - 2[-\sin x] = \sqrt{3} + 2\sin x$.

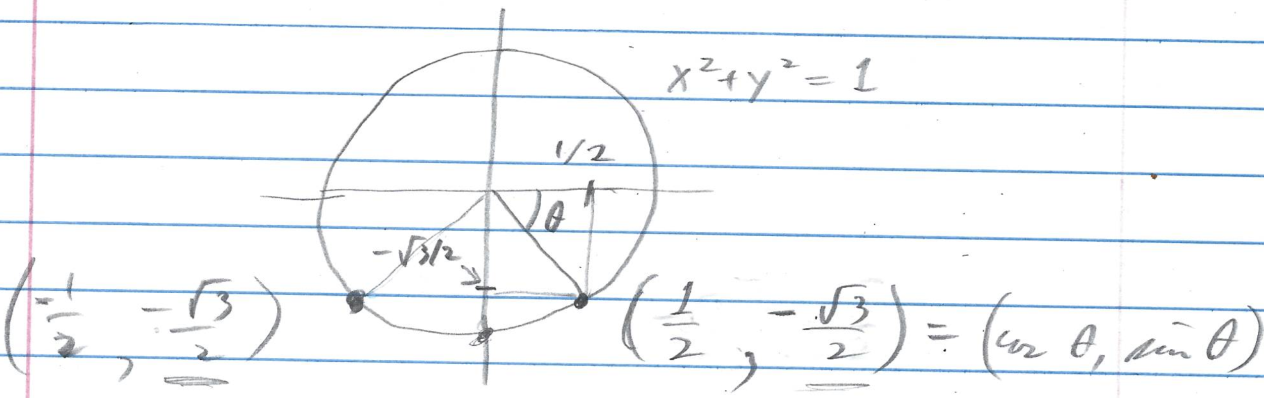
Consider the equation

$$f'(x) = \sqrt{3} + 2\sin x := 0 \quad \text{for } x \in [0, 2\pi]$$

$$\text{OR} \qquad \sin x = -\frac{\sqrt{3}}{2} \quad \text{for } x \in [0, 2\pi].$$

Now, $\sin^{-1}(-\sqrt{3}/2) = -\pi/3$ (from a calculator).

Consider the unit circle:



So, for $\sin x = \frac{-\sqrt{3}}{2}$ AND $x \in [0, 2\pi]$ we need $x = \frac{4\pi}{3}$ or $x = \frac{5\pi}{3}$.

So, f has critical points of $x = 4\pi/3$ and $x = 5\pi/3$ since $f' = 0$ there.

So consider

	$[0, 4\pi/3)$	$(4\pi/3, 5\pi/3)$	$(5\pi/3, 2\pi]$
TEST VALUE h	0	$3\pi/2$	2π
$f'(h)$	$\sqrt{3}+2\sin(0)$ $=\sqrt{3}$	$\sqrt{3}+2\sin\left(\frac{3\pi}{2}\right)$ $=\sqrt{3}+2(-1)$	$\sqrt{3}+2\sin(2\pi)$ $=\sqrt{3}+2(0)=\sqrt{3}$
$f'(x)$	$+$	$-$	$+$
$f(x)$	INC	DEC	INC

SO, by the First Derivative Test for Extrema (Theorem) implies that f(x) has a local MAX at $x=4\pi/3$ of

$$f(4\pi/3) = \sqrt{3}(4\pi/3) - 2\cos(4\pi/3)$$
$$= 4\sqrt{3}\pi/3 - 2\left(-\tfrac{1}{2}\right)$$
$$= 4\pi/\sqrt{3} + 1,$$

AND f has a local MIN at $x=5\pi/3$ of

$$f(5\pi/3) = \sqrt{3}(5\pi/3) - 2\cos(5\pi/3)$$
$$= 5\sqrt{3}\pi/3 - 2\left(\tfrac{1}{2}\right)$$
$$= 5\pi/\sqrt{3} - 1.$$

Next, for concavity consider

$$f''(x) = \frac{d}{dx}\left[\sqrt{3}+2\sin x\right] = 2\cos(x).$$

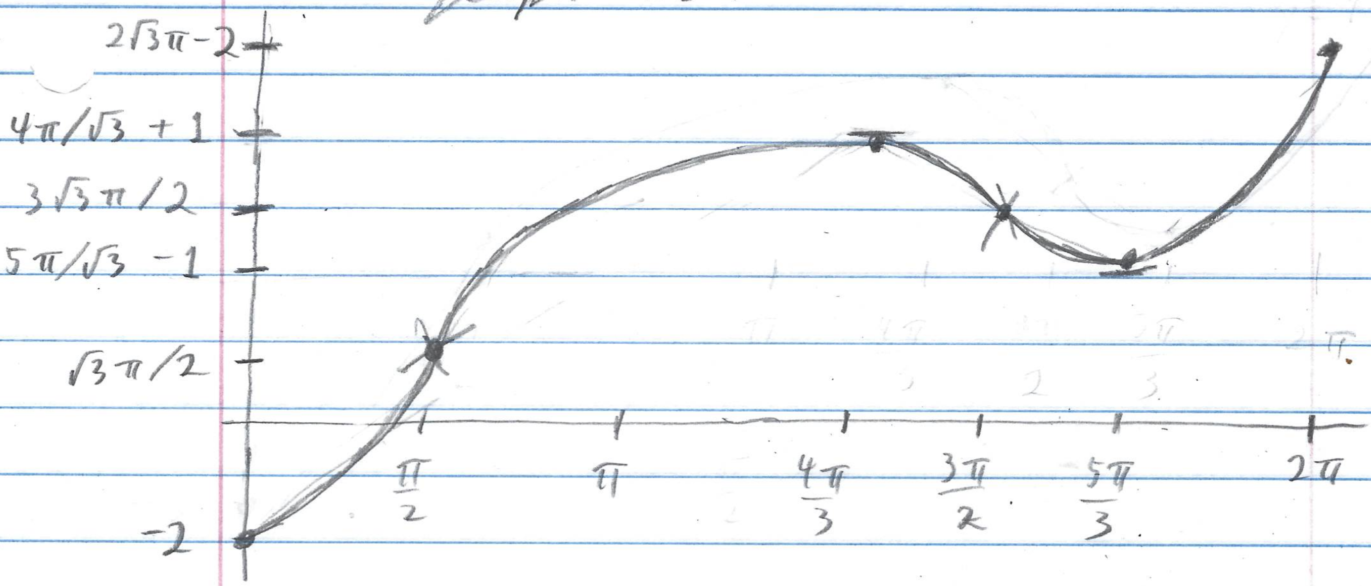
Now $f''(x) = 2\cos(x) := 0$ and $x \in [0, 2\pi]$ implies $\cos(x)=0$ or $x=\pi/2$ or $x=3\pi/2$ are potential points of inflection.

So consider

	$[0, \pi/2)$	$(\pi/2, 3\pi/2)$	$(3\pi/2, 2\pi]$
TEST VALUE k	0	π	2π
$f''(k)$	$2\cos(0) = 2$	$2\cos(\pi) = -2$	$2\cos(2\pi) = 2$
$f''(x)$	$+$	$-$	$+$
$f(x)$	CU	CD	CU

SO $x = \pi/2$ and $x = 3\pi/2$ really are points of inflection.

The graph is:



(Notice $f(0) = \sqrt{3}(0) - 2\cos(0) = -2$,
$f(\pi/2) = \sqrt{3}(\pi/2) - 2\cos(\pi/2) = \sqrt{3}\pi/2$,
$f(3\pi/2) = \sqrt{3}(3\pi/2) - 2\cos(3\pi/2) = 3\sqrt{3}\pi/2$,
$f(2\pi) = \sqrt{3}(2\pi) - 2\cos(2\pi) = 2\sqrt{3}\pi - 2$.)

So absolute MAX is $2\sqrt{3}\pi - 2$ at $x = 2\pi$ and the absolute MIN is -2 at $x = 0$.

$\square$