4.4.95 Consider $y = f(x) = \dfrac{x^2 - x + 1}{x - 1}$. Apply the seven steps in the "Procedure for Graphing."

Solution

① The domain of f is $(-\infty, 1) \cup (1, \infty)$. Since $f(x) \neq f(-x)$ then f is not symmetric with respect to the y-axis. Since $f(x) \neq -f(-x)$ then f is not symmetric with respect to the origin.

② We have

$$y' = f'(x) = \frac{[2x-1](x-1) - (x^2 - x + 1)[1]}{(x-1)^2}$$

$$= \frac{2x^2 - 3x + 1 - x^2 + x - 1}{(x-1)^2} = \frac{x^2 - 2x}{(x-1)^2} = \frac{x(x-2)}{(x-1)^2},$$

and $y'' = f''(x) = \dfrac{[2x-2](x-1)^2 - (x^2 - 2x)[2(x-1)[1]]}{((x-1)^2)^2}$

$$= \frac{2(x-1)\left((x-1)^2 - (x^2 - 2x)\right)}{(x-1)^4} = \frac{2\left((x^2 - 2x + 1) - x^2 + 2x\right)}{(x-1)^3}$$

$$= \frac{2}{(x-1)^3}.$$

③ Since $f'(x) = \dfrac{x(x-2)}{(x-1)^2}$ then $x = 0$ and $x = 2$ are critical points since $f' = 0$ there.

④ We consider

	$(-\infty, 0)$	$(0, 1)$	$(1, 2)$	$(2, \infty)$
TEST VALUE k	-1	$1/2$	$3/2$	3
$f'(k)$	$\dfrac{(-1)((-1)-2)}{((-1)-1)^2}$	$\dfrac{(1/2)((1/2)-2)}{((1/2)-1)^2}$	$\dfrac{(3/2)((3/2)-2)}{((3/2)-1)^2}$	$\dfrac{(3)((3)-2)}{((3)-1)^2}$
$f'(x)$	$+$	$-$	$-$	$+$
$f(x)$	INC	DEC	DEC	INC

So f is INC on $(-\infty, 0] \cup (1, 2]$ and
f is DEC on $[0, 1) \cup [2, \infty)$.

(5) Since $f''(x) = 2/(x-1)^3$ then there are no points of inflection. We consider

	$(-\infty, 1)$	$(1, \infty)$
TEST VALUE k	0	2
$f''(k)$	$2/((0)-1)^3$	$2/((2)-1)^3$
$f''(x)$	$-$	$+$
$f(x)$	CD	CU

So f is CD on $(-\infty, 1)$ and f is CU on $(1, \infty)$.

(6) Since $f(x) = \dfrac{x^2 - x + 1}{x - 1}$ then by Dr. Bob's definite Limits Theorem, $y = f(x)$ has a vertical asymptote at $x = 1$, $\lim\limits_{x \to 1^-} f(x) = \pm\infty$, and $\lim\limits_{x \to 1^+} f(x) = \pm\infty$. For $x \to 1^-$, $\dfrac{x^2 - x + 1}{x - 1} \Rightarrow \dfrac{(+)}{(-)} = -$ and so $\lim\limits_{x \to 1^-} f(x) = -\infty$. For $x \to 1^+$,

$\dfrac{x^2 - x + 1}{x - 1} \Rightarrow \dfrac{(+)}{(+)} = +$ and so $\lim\limits_{x \to 1^+} f(x) = \infty$.

Since f is a rational function and the degree of the numerator is one more than the degree of the denominator, then $y = f(x)$ has (by definition) an oblique asymptote.

We have $\begin{array}{r} x \\ (x-1)\overline{)\,x^2-x+1} \\ \underline{-(x^2-x)} \\ 1 \end{array}$, so $f(x) = x + \dfrac{1}{x-1}$,

and $y = x$ is an oblique asymptote of $y = f(x)$.

(7) Notice that $f(x) = \dfrac{x^2-x+1}{x-1} = 0$ implies

$x^2 - x + 1 = 0$ or $x = \dfrac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(1)}}{2(1)}$

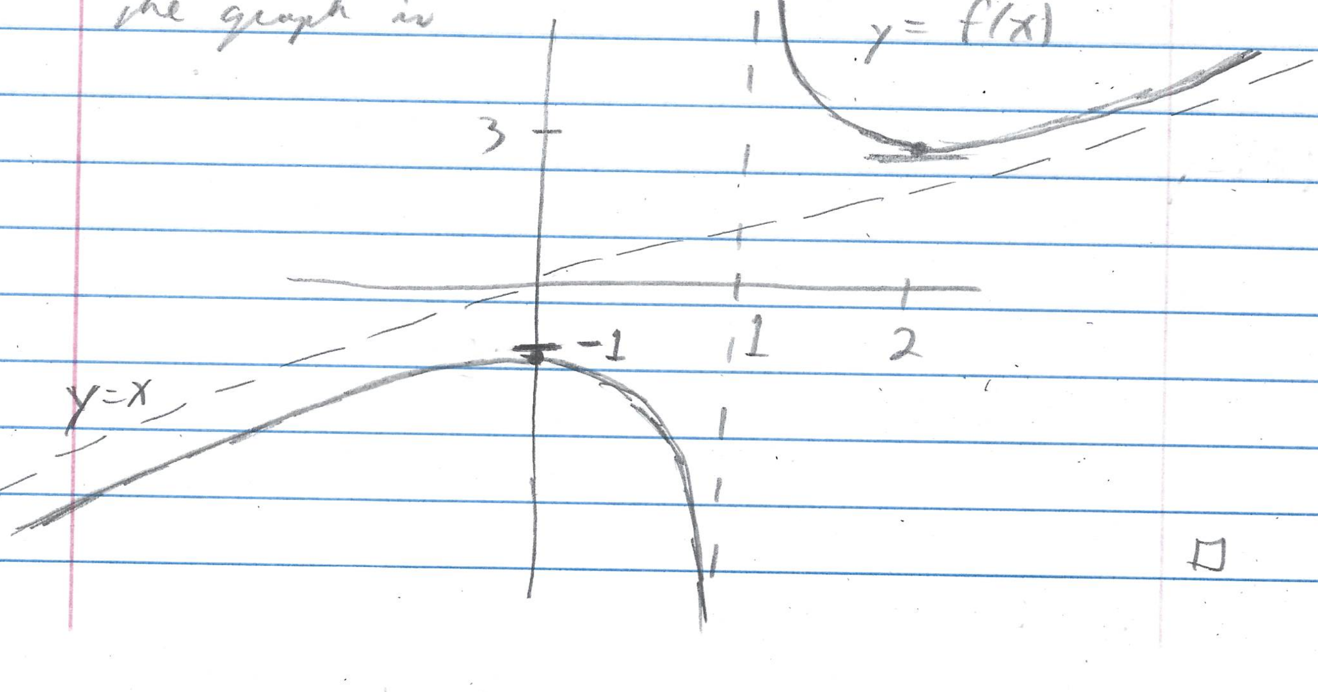
or $x = \dfrac{1 \pm \sqrt{-3}}{2}$, so $y = f(x)$ has no (real) x-intercept.

We have $f(0) = \dfrac{(0)^2 - (0) + 1}{(0) - 1} = -1$

(this is the y-intercept and a local MAX by the First Derivative Test, Theorem 4.3.1).

Also $f(2) = \dfrac{(2)^2 - (2) + 1}{(2) - 1} = 3$ (this is a local MAX by the First Derivative Test).

The graph is



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