4.5.31 Evaluate $\lim_{x \to \infty} \frac{\ln(x+1)}{\log_2(x)}$.

Solution

Recall from the graph of logarithm functions that $\lim_{x\to\infty} \log_a(x) = \infty$ for $a > 1$.

So $\lim_{x\to\infty} \ln(x+1) = \infty$ and $\lim_{x\to\infty} \log_2(x) = \infty$. So the given limit is of the ∞/∞ indeterminant form. Hence, by l'Hopital's Rule (Theorem 4.5.A) we have

$$\lim_{x\to\infty} \frac{\ln(x+1)}{\log_2(x)} \overset{\infty/\infty}{=} \lim_{x\to\infty} \frac{\frac{1}{(x+1)}}{\frac{1}{\ln 2}\left(\frac{1}{x}\right)}$$

since $\frac{d}{dx}\left[\log_a(x)\right] = \frac{1}{\ln a}\frac{1}{x}$

$$= \ln(2) \lim_{x\to\infty}\left(\frac{x}{x+1}\right)$$

$$\overset{\infty/\infty}{=} \ln(2) \lim_{x\to\infty} \frac{(1)}{(1)} = \boxed{\ln(2)}. \quad \square$$