

4.5.73 Consider  $\lim_{x \rightarrow \infty} \frac{e^{x^2}}{xe^x}$ . L'Hopital's Rule

does not help with this limit — it just keeps cycling back to the original limit. Find the limit in some other way.

Solution

We have

$$\lim_{x \rightarrow \infty} \frac{e^{x^2}}{xe^x} = \lim_{x \rightarrow \infty} \frac{e^{x^2-x}}{x}$$

$$\frac{\infty}{\infty} \lim_{x \rightarrow \infty} \frac{e^{x^2-x} \rightarrow [2x-1]}{1}$$

$$= \lim_{x \rightarrow \infty} (2x-1) e^{x^2-x}$$

$$= \boxed{\infty} \text{ since } \lim_{x \rightarrow \infty} (2x-1) = \infty \text{ and}$$

$$\lim_{x \rightarrow \infty} e^{x^2-x} = \infty. \quad \square$$