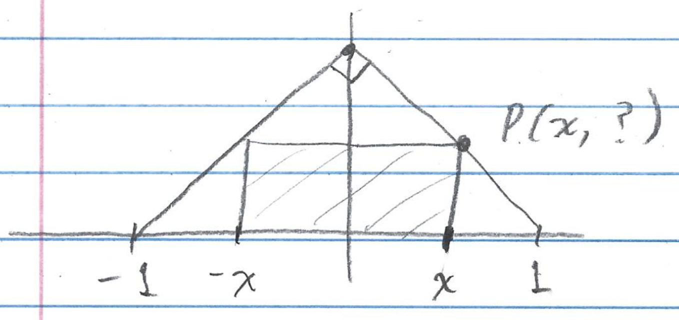
4.6.3 Consider the figure:

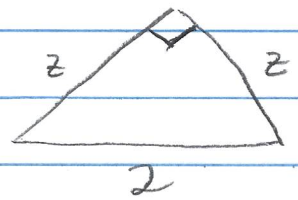


What is the area of the largest (area) rectangle that can be inscribed in the triangle?

Solution

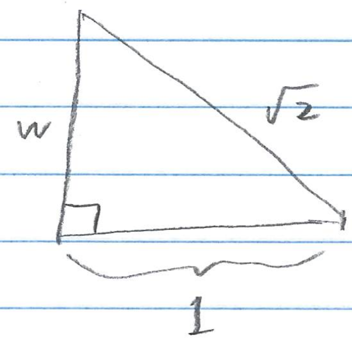
(2) and (3) We have the above picture. Notice the base of the rectangle is $x-(-x) = 2x$. Let the height be y.

Notice by the Pythagorean Theorem:



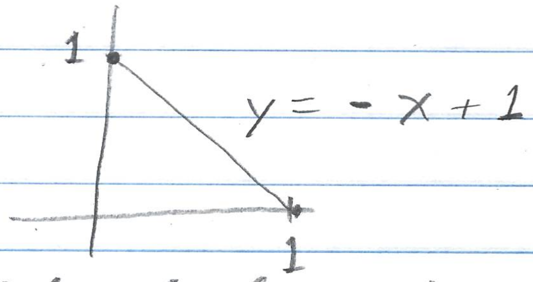
$$z^2 + z^2 = (2)^2$$
$$2z^2 = 4$$
$$z^2 = 2$$
$$z = \sqrt{2} \quad (z \geq 0).$$

Again, by the Pythagorean Theorem



$$w^2 + (1)^2 = (\sqrt{2})^2$$
$$w^2 + 1 = 2$$
$$w^2 = 1$$
$$w = 1 \quad (w \geq 0).$$

So, we have



Hence, the height of the rectangle is $y = -x + 1$.

(4) The area of the rectangle is

$$A = (\text{base})(\text{height}) = (2x)y$$
$$= 2x(-x+1) = A(x).$$

(5) The question is: "What is the MAX of $A(x) = 2x(-x+1)$ for $x \in [0,1]$.
Since $A(x) = -2x^2 + 2x$ then:
$A'(x) = -4x + 2$ and we have a critical point $-4x + 2 = 0$ or $x = 1/2$.
So we consider A at the endpoints 0 and 1 and the critical point $x = 1/2$:

x	0	$1/2$	1
$A(x)$	0	$-2(\frac{1}{2})^2 + 2(\frac{1}{2}) = 1/2$	0

Hence, the maximum area is $1/2$ and occurs when $x = 1/2$ and $y = 1/2$, so the rectangle has width $2x = 1$ and height $y = 1/2$.

$\square$