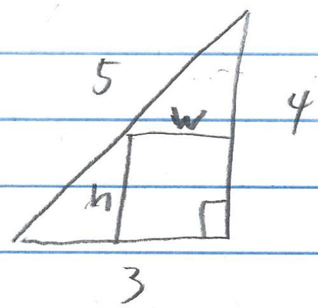
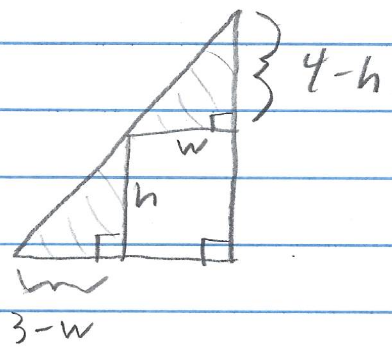
4.6.33 What is the area of largest area rectangle that can be inscribed in the given triangle?




Solution

(2) and (3) We are given the picture.
The rectangle has width $w$ and height $h$.
So the area of the rectangle is $\underline{A = wh}$.
We need a relationship between $w$ and $h$.

We have similar triangles:




so,

$$\frac{4-h}{w} = \frac{h}{3-w}$$

or $(4-h)(3-w) = hw$

or $12 - 3h - 4w + hw = hw$

or $12 - 3h - 4w = 0$ or $3h = 12 - 4w$

or $h = \dfrac{12-4w}{3} = 4 - \dfrac{4}{3}w.$

(4) The area then is

$$A = wh = w\left(4 - \frac{4}{3}w\right) = A(w).$$

The question is, what is the MAX of $A(w)$ for $w \in [0,3]$.

(5) To maximize $A(w)$ consider:

$A(w) = 4w - \frac{4}{3}w^2$ and $A'(x) = 4 - \frac{8}{3}w$

and we have a critical point when

$4 - \frac{8}{3}w = 0$ or $\frac{8}{3}w = 4$ or $w = 4\left(\frac{3}{8}\right) = \frac{3}{2}$.

SO, we consider

| $w$ | $0$ | $3/2$ | $3$ |
|---|---|---|---|
| $A(w)$ | $0$ | $4\left(\frac{3}{2}\right) - \frac{4}{3}\left(\frac{3}{2}\right)^2 = 6 - 3 = 3$ | $0$ |

$\boxed{\text{Hence area is a MAX of } 3 \text{ and occurs when } w = 3/2 \text{ and } h = 4 - \frac{4}{3}\left(\frac{3}{2}\right) = 2.}$

$\square$