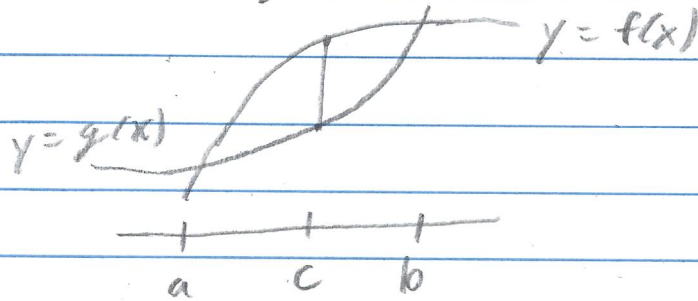**4.6.69** Let $f(x)$ and $g(x)$ be the differentiable functions graphed below. Point c is the point where the vertical distance between the curves is the greatest. Is there anything special about the tangents to the two curves at c?

Solution

② and ③ We have the picture above and the distance between the curves is a function of x is $D(x) = f(x) - g(x)$ where $x \in [a, b]$.

④/⑤ To maximize D on $[a, b]$ we consider $D'(x) = f'(x) - g'(x)$. Since f and g are differentiable, then the critical points of D result from $D'(x) = f'(x) - g'(x) = 0$.

So we must have $D'(c) = f'(c) - g'(c) = 0$ or $f'(c) = g'(c)$. So at the maximum distance apart, the slope of a tangent to f is the same as the slope of a tangent to g. ☐