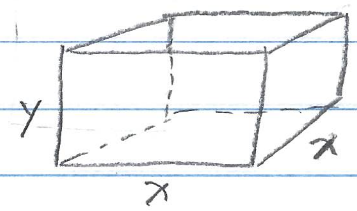
4.6.9 Your iron works has contracted to design and build a 500 ft^3, square-based, open-top, rectangular steel holding tank for a paper company. The tank is made by welding thin stainless steel plates together along their edges. As the production engineer, your job is to find dimensions for the base and height that will make the tank weigh as little as possible. What dimensions do you tell the shop to use?

Solution

(2) and (3) We have



We know the volume is $V = 500$ ft^3. The surface area (i.e., the amount of steel sheeting used) is

$$A = (\text{base}) + 4(\text{sides}) = x^2 + 4(xy).$$

Also the volume is

$$V = 500 = x^2 y \text{ and so } y = \frac{500}{x^2}.$$

(4) The question is what is MIN area (to minimize weight) where

$$A = x^2 + 4xy = x^2 + 4x\left(\frac{500}{x^2}\right)$$

$$= x^2 + \frac{2000}{x} = A(x) \text{ for } x \in (0, \infty).$$

(5) We have $A'(x) = 2x - \frac{2000}{x^2} = \frac{2x^3 - 2000}{x^2}$,

so there is a critical point when $2x^3 - 2000 = 0$

or $x^3 = 1000$ or $x = \sqrt[3]{1000} = 10$ ft.

We consider A' on $(0, \infty)$:

	$(0, 10)$	$(10, \infty)$
TEST VALUE h	1	20
$A'(h)$	$\frac{2(1)^3 - 2000}{(1)^2} = -1998$	$\frac{2(20)^3 - 2000}{(20)^2} = \frac{14,000}{400}$
$A'(x)$	$-$	$+$
$A(x)$	DEC	INC

So by the First Derivative Test (Theorem 4.3.A),

A has a local MIN at $x = 10$ ft of

$$A(10) = (10)^2 + \frac{2000}{(10)} = 100 + 200 = 300 \text{ ft}^3.$$

Since there is only one critical point of A in $(0, \infty)$, we see that the local MIN is in fact an absolute MIN.

$\boxed{\text{So the minimum area (and hence minimum weight) of } 300 \text{ ft}^3 \text{ occurs when the base has width and depth } x = 10 \text{ ft and height } y = \frac{500}{(10)^2} = 5 \text{ ft.}}$

$\square$