4.7.1 Use Newton's Method to estimate the solutions of the equation $x^2+x-1=0$. Start with $x_0=-1$ for the left-hand solution and with $x_0=1$ for the solution on the right. Then, in each case, find x_2.

Solution

First, we set $f(x)=x^2+x-1$ so that we can apply Newton's Method to the equation $f(x)=0$, as needed. We then have $f'(x)=2x+1$. We make a table for each value of x_0.

n	x_n	$f(x_n)$	$f'(x_n)$	$x_{n+1}=x_n-f(x_n)/f'(x_n)$
0	-1	$(-1)^2+(-1)-1$ $=-1$	$2(-1)+1=-1$	$(-1)-(-1)/(-1)=-2$
1	-2	$(-2)^2+(-2)-1$ $=1$	$2(-2)+1=-3$	$-2-(1)/(-3)$ $=-2+\frac{1}{3}=-5/3$

n	x_n	$f(x_n)$	$f'(x_n)$	$x_{n+1}=x_n-f(x_n)/f'(x_n)$
0	1	$(1)^2+(1)-1$ $=1$	$2(1)+1=3$	$(1)-(1)/(3)=2/3$
1	$2/3$	$\left(\frac{2}{3}\right)^2+\left(\frac{2}{3}\right)-1$ $=\frac{4}{9}+\frac{2}{3}-1=\frac{1}{9}$	$2\left(\frac{2}{3}\right)+1$ $=\frac{7}{3}$	$\left(\frac{2}{3}\right)-\left(\frac{1}{9}\right)/\left(\frac{7}{3}\right)$ $=\frac{2}{3}-\frac{1}{21}=\frac{13}{21}$

So with $x_0=-1$ we get $x_2=-5/3$ and with $x_0=1$ we get $x_2=13/21$.

$\square$