

4.7.5 Use Newton's Method to find the positive fourth root of 2 by solving the equation $x^4 - 2 = 0$. Start with $x_0 = 1$ and find x_2 .

Solution

First, we set $f(x) = x^4 - 2$ so that we can apply Newton's Method to the equation $f(x) = 0$, as needed. We then have $f'(x) = 4x^3$.

We make a table of relevant values:

n	x_n	$f(x_n)$	$f'(x_n)$	$x_{n+1} = x_n - f(x_n)/f'(x_n)$
0	1	$(1)^4 - 2 = -1$	$4(1)^3 = 4$	$1 - (-1)/4 = 5/4$
1	$5/4$	$(5/4)^4 - 2 =$ $\frac{625}{256} - \frac{512}{256}$ $= 113/256$	$4(5/4)^3 =$ $\frac{500}{64} = \frac{125}{16}$	$\frac{5}{4} - \frac{113/256}{125/16}$ $= \frac{5}{4} - \frac{113/16}{125}$

$$\text{So } x_2 = \frac{5}{4} - \frac{113/16}{125} = \frac{5}{4} - \frac{113}{2000} = \frac{2500}{2000} - \frac{113}{2000}$$

$$= \frac{2387}{2000} = 1.1935$$

Notice that, to four decimal places, $\sqrt[4]{2} \approx 1.1892$. \square