4.8.97 Solve the initial value problem;

$$\frac{ds}{dt} = 1 + \cos(t) \quad \text{and} \quad S(0) = 4.$$

Solution.

We need an antiderivative of $\frac{ds}{dt}$. So

$$S(t) \in \int \left(\frac{ds}{dt}\right) dt$$

$$= \int \left(1 + \cos(t)\right) dt$$

$$= t + \sin(t) + C.$$

Hence $S(t) = t + \sin(t) + k$ where k is some constant. Using the initial condition $S(0) = 4$ we need

$$S(0) = (0) + \sin(0) + k \equiv 4, \text{ and}$$

so $k = 4$.

Therefore $\boxed{S(t) = t + \sin(t) + 4.}$ □