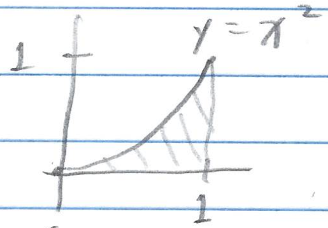
5.1.5 Using rectangles each of whose height is given by the values of the function at the midpoint of the rectangle's base ("the midpoint rule"), estimate the area under the graph of $f(x) = x^2$ between $x = 0$ and $x = 1$ using first two and then four rectangles.

<u>Solution</u>

The area is:



Using two subintervals, $[0, 1/2]$ ad $[1/2, 1]$, and corresponding midpoints $c_1 = 1/4$ ad $c_2 = 3/4$, we have

$$(\text{area}) \simeq f(c_1)\Delta x_1 + f(c_2)\Delta x_2$$

$$= f\left(\tfrac{1}{4}\right)\left(\tfrac{1}{2} - 0\right) + f\left(\tfrac{3}{4}\right)\left(1 - \tfrac{1}{2}\right)$$

$$= \left(\tfrac{1}{4}\right)^2\left(\tfrac{1}{2}\right) + \left(\tfrac{3}{4}\right)^2\left(\tfrac{1}{2}\right)$$

$$= \frac{1}{32} + \frac{9}{32} = \frac{10}{32} = \boxed{\frac{5}{16}}$$

Using four subintervals, $[0, 1/4]$, $[1/4, 1/2]$, $[1/2, 3/4]$, and $[3/4, 1]$, and corresponding midpoints $c_1 = 1/8$, $c_2 = 3/8$, $c_3 = 5/8$, and $c_4 = 7/8$, we have

$$(\text{area}) \approx f(c_1)\Delta x_1 + f(c_2)\Delta x_2 + f(c_3)\Delta x_3 + f(c_4)\Delta x_4$$

$$= f\left(\tfrac{1}{8}\right)\left(\tfrac{1}{4} - 0\right) + f\left(\tfrac{3}{8}\right)\left(\tfrac{1}{2} - \tfrac{1}{4}\right) + f\left(\tfrac{5}{8}\right)\left(\tfrac{3}{4} - \tfrac{1}{2}\right)$$

$$+ f\left(\tfrac{7}{8}\right)\left(1 - \tfrac{3}{4}\right)$$

$$= \left(\tfrac{1}{8}\right)^2\left(\tfrac{1}{4}\right) + \left(\tfrac{3}{8}\right)^2\left(\tfrac{1}{4}\right) + \left(\tfrac{5}{8}\right)^2\left(\tfrac{1}{4}\right) + \left(\tfrac{7}{8}\right)^2\left(\tfrac{1}{4}\right)$$

$$= \frac{1}{256} + \frac{9}{256} + \frac{25}{256} + \frac{49}{256} = \frac{84}{256} = \boxed{\frac{21}{64}}. \quad \square$$