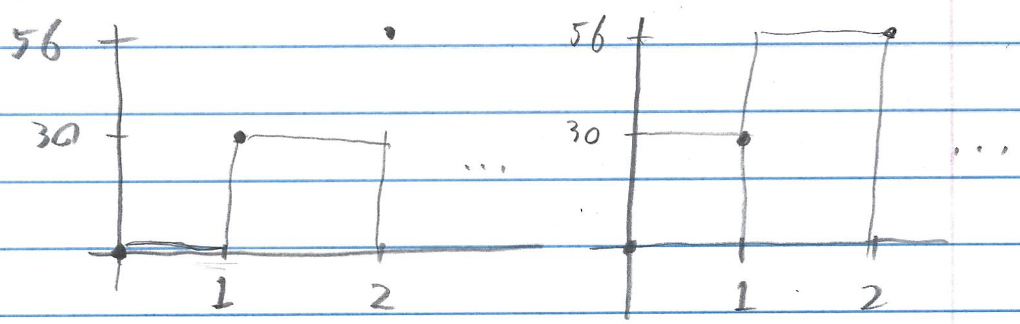
5.1.9 The accompanying table shows the velocity of a model train engine moving along a track for 10 sec. Estimate the distance traveled by the engine using 10 subintervals of length 1 with (a) left-endpoint values, and (b) right-endpoint values.

Time (sec)	Velocity (cm/sec)	Time (sec)	Velocity (cm/sec)
0	0	6	28
1	30	7	15
2	56	8	5
3	25	9	15
4	38	10	0
5	33		

Solution

The idea is to graph the velocity function over the interval [0, 10] and use (a) left-hand endpoints and (b) right-hand endpoints to estimate the area under the velocity function; notice these areas correspond to distance traveled:



(a) (b)

(a) So using left-hand endpoints (and observing that the subintervals are all of width 1 sec) we have

$$\begin{pmatrix}\text{distance}\\ \text{traveled}\end{pmatrix} \approx (0\text{ cm/sec})(1\text{ sec}) + (30\text{ cm/sec})(1\text{ sec})$$

$$+ (56\text{ cm/sec})(1\text{ sec}) + (25\text{ cm/sec})(1\text{ sec})$$
$$+ \cdots + (5\text{ cm/sec})(1\text{ sec}) + (15\text{ cm/sec})(1\text{ sec})$$

$$= (0 + 30 + 56 + 25 + 38 + 33 + 28 + 15 + 5 + 15)\text{ cm}$$

$$= \boxed{245\text{ cm.}}$$

(b) Using right-hand endpoints we have

$$\begin{pmatrix}\text{distance}\\ \text{traveled}\end{pmatrix} \approx (30\text{ cm/sec})(1\text{ sec}) + (56\text{ cm/sec})(1\text{ sec})$$
$$+ \cdots + (15\text{ cm/sec})(1\text{ sec}) + (0\text{ cm/sec})(1\text{ sec})$$

$$= (30 + 56 + 25 + 38 + 33 + 28 + 15 + 5 + 15 + 0)\text{ cm}$$

$$= \boxed{245\text{ cm.}} \quad \square$$