

5.4.77 Find an antiderivative of the integrand and evaluate $\int_2^5 \frac{x}{\sqrt{1+x^2}} dx$.

Solution

Well, $f(x) = \frac{x}{\sqrt{1+x^2}} = x(1+x^2)^{-1/2}$.
UH... let's try $(1+x^2)^{1/2}$ for an antiderivative:

$$\begin{aligned} \frac{d}{dx} [(1+x^2)^{1/2}] &= \frac{1}{2} (1+x^2)^{-1/2} [2x] \\ &= x(1+x^2)^{-1/2}. \quad \text{😊} \end{aligned}$$

So, by the Fundamental Theorem of Calculus, Part 2,

$$\begin{aligned} \int_2^5 \frac{x}{\sqrt{1+x^2}} dx &= \left(\sqrt{1+x^2} \right) \Big|_2^5 \\ &= \sqrt{1+(5)^2} - \sqrt{1+(2)^2} = \boxed{\sqrt{26} - \sqrt{5}}. \quad \square \end{aligned}$$