5.4.45 Find dy/dx for $y = \int_0^x \sqrt{1+t^2}\, dt$.

Solution

Notice that $f(t) = \sqrt{1+t^2}$ is continuous on all of $\mathbb{R}$, so by the Fundamental Theorem of Calculus, Part 1,

$$\frac{dy}{dx} = \frac{d}{dx}\left[\int_0^x f(t)\, dt\right] = f(x) \text{ or}$$

$$\frac{dy}{dx} = \frac{d}{dx}\left[\int_0^x \sqrt{1+t^2}\, dt\right] = \boxed{\sqrt{1+x^2}.} \quad \square$$