LIKE

5.4.53 Find $dy/dx$ for $y = \int_1^{e^{x^2}} \frac{1}{\sqrt{t}}\,dt$.

Solution

Notice that the integrand $\frac{1}{\sqrt{t}}$ is continuous on $[1, e^{x^2}]$ (since $e^{x^2} \geq 1$), so we can apply the Fundamental Theorem of Calculus, Part 1. Let $u = u(x) = e^{x^2}$ and apply the Chain Rule: $\frac{dy}{dx} = \frac{dy}{du}\left[\frac{du}{dx}\right]$.

So

$$\frac{dy}{dx} = \frac{d}{du}\left[\int_1^u \frac{1}{\sqrt{t}}\,dt\right]\left[\frac{du}{dx}\right]$$

$$= \frac{d}{du}\left[\int_1^u \frac{1}{\sqrt{t}}\,dt\right]\frac{d}{dx}\left[e^{x^2}\right]$$

$$= \frac{1}{\sqrt{u}}\left[e^{x^2}[2x]\right]$$

$$= \frac{1}{\sqrt{e^{x^2}}}\,2x\,e^{x^2} = \frac{2x\,e^{x^2}}{e^{x^2/2}}$$

$$= \boxed{2x\,e^{x^2/2}}. \quad \square$$