

9.5.13 Evaluate  $\int \sqrt{x} \sin^2(x^{3/2} - 1) dx$ .

Solution

Well,

$$\int \boxed{x^{1/2} \sin^2(x^{3/2} - 1)} dx$$

$$\text{let } u = x^{3/2} - 1$$

$$du = \frac{3}{2} x^{1/2} dx, \text{ or}$$

$$\frac{2}{3} du = x^{1/2} dx$$

$$= \int \sin^2(u) \frac{2}{3} du = \frac{2}{3} \int \sin^2(u) du.$$

Recall  $\sin^2 \theta = \frac{1 - \cos(2\theta)}{2}$

$$\cos^2 \theta = \frac{1 + \cos(2\theta)}{2} \quad (!)$$

$$\text{So, } \frac{2}{3} \int \sin^2 u \, du = \frac{2}{3} \int \frac{1 - \cos(2u)}{2} \, du$$

$$= \frac{2}{3} \int \left( \frac{1}{2} - \frac{1}{2} \cos(2u) \right) du.$$

$$= \frac{2}{3} \left( \frac{1}{2} u - \frac{1}{2} \left( \frac{1}{2} \sin(2u) \right) \right) + C$$

$$= \boxed{\frac{1}{3} (x^{3/2} - 1) - \frac{1}{6} \sin(2(x^{3/2} - 1))} + C. \quad \square$$