

5.5.41

$$\int \sqrt{\frac{x^3-3}{x^8}} dx = ?$$

Solution This one is tricky! We have

$$\int \sqrt{\frac{x^3-3}{x^8}} dx = \int \sqrt{\frac{x^3-3}{x^8 x^3}} dx = \int \frac{1}{x^4} \sqrt{\frac{x^3-3}{x^3}} dx$$

$$= \int x^{-4} \sqrt{1-3x^{-3}} dx \quad \text{let } u = 1-3x^{-3}$$

$$du = 9x^{-4} dx \text{ or}$$

$$du/9 = x^{-4} dx$$

$$= \int \sqrt{u} du/9 = \frac{1}{9} \int u^{1/2} du$$

$$= \frac{1}{9} \left( \frac{2}{3} u^{3/2} \right) + C = \frac{2}{27} (1-3x^{-3})^{3/2} + C$$

$$= \boxed{\frac{2}{27} \left( 1 - \frac{3}{x^3} \right)^{3/2} + C} \quad \square$$