

5.5.45

$$\int (x+1)^2 (1-x)^5 dx = ?$$

Solution

Instead of multiplying out, we'll let $u = 1-x$ (since it is raised to the highest power) to get

$$\int (x+1)^2 (1-x)^5 dx$$

let $u = 1-x$,
 $du = -dx$ or $-du = dx$
 and $x = 1-u$

$$= \int ((1-u)+1)^2 u^5 (-du) = - \int (2-u)^2 u^5 du$$

$$= - \int (4 - 4u + u^2) u^5 du = - \int (4u^5 - 4u^6 + u^7) du$$

$$= - \left(\frac{4}{6} u^6 - \frac{4}{7} u^7 + \frac{1}{8} u^8 \right) + C$$

$$= \boxed{-\frac{2}{3} (1-x)^6 + \frac{4}{7} (1-x)^7 - \frac{1}{8} (1-x)^8 + C} \quad \square$$